Frege and truth-values as references

Abílio Rodrigues

Universidade Federal de Minas Gerais

abilio@ufmg.br

Abstract

The aim of this paper is to show that Frege's argument that concluded that the reference of a sentence is its truth-value, presented in *On sense and reference* (1892), may be reconstructed taking into account the problems of the notion of conceptual content presented in the *Begriffsschrift* (1879) and also other passages from a letter to Russell (1902) and the posthumous *Logic in Mathematics* (1914). I claim that the reconstructed argument is perfectly sound and convincing.

1. Introduction

The thesis that the reference of a sentence is its truth-value is a central point of Frege's philosophical work. It was criticized maybe for the first time by Russell in a letter to Frege. Russell objected that he cannot

> believe that the true or the false is the reference of a proposition in the same sense as, e.g., a certain person is the reference[1] of the name Julius Caesar. (Frege, 1980 pp. 150-1).

Russell's view is somewhat similar to Dummett's (1973) and Chateaubriand's (2001) views. According to Dummett, we are justified in taking the relation between a proper name and its bearer as the prototype for the relation between a sentence and its reference, and so he argues that Frege should not have ascribed reference to sentences (Dummett 1973, p. 181).

---

[1] In Frege (1980) we read 'meaning' instead of 'reference'. The discussion about the translation of 'Bedeutung' is well-known. I have changed here all the occurrences of 'meaning' rendering 'Bedeutung' by 'reference', in quotations from Frege 1960, 1979 and 1980.

At this point, Dummett is doubly mistaken: the prototype *is not* the relation between a name and its bearer, and of course Frege *had to* ascribe reference (i.e. semantic value) to the sentences of his formal language. Some pages later, we read:

> The identification of truth-values as referents of sentences, taken together with the thesis that the truth-values are objects, led to a great simplification of Frege's ontology, at the price of a highly implausible analysis of language … It is tragic that a thinker who achieved the first really penetrating analysis of the structure of our language should have found himself driven to such absurdities … the assimilation of sentences to proper names did have a fatal effect upon Frege's theory of meaning. It is just that Frege's earlier departures from the forms of natural language … were founded upon deep insights into the workings of language; whereas this ludicrous deviation is prompted by no necessity, but is a gratuitous blunder (Dummett, 1973, p. 184)

As it is well-known, Dummett reads Frege from the viewpoint of his (Dummett's) interest in a theory of meaning. Frege, however, was not interested in a theory of meaning; he was interested in devising a theory of logical consequence in order to carry out his logicist project. Notice, besides, that it is perfectly feasible to understand Frege's claim that truth-values are objects as pragmatically motivated, similar to our practice of writing on the blackboard things like $I(p) = T$, $I(q) = F$, so $I(p \land q) = F$ when teaching propositional logic, and things like $I(a) = Aristotle$ in teaching predicate logic – we treat sentences as names and truth-values as objects. Chateaubriand also criticizes Frege's choice, in my view, committing the same error as Dummett:

> [I]f one looks for candidates, one will hit upon states of affairs, facts, or something like that. The similarity between a definite description, as a way of presenting an object, and a sentence, as a way of presenting a state of affairs, is so striking and obvious … that it just can't be missed. (Chateaubriand, 2001 p. 76).

However, it is not unfair to say that Frege has *discovered* that the extension of a sentence is its truth-value and that an extensional logic is very suitable as an account of logical

consequence for Mathematics. The fact that Frege explained informally the notion of reference starting from the puzzle of identity and based on examples of natural language, where the relation between a singular term and its reference is that of a name and its bearer, should not be a problem. Frege's strategy may be understood as an informal and pre-theoretical elucidation, in Weiner's sense (Weiner 2001). The relation between a sentence and a truth-value is not like the relation between a name and its bearer, rather, it is a relation between a linguistic expression and its semantic value within a formal system. The relation between a singular term and the corresponding object is also a relation between an expression and its semantic value, although it happens that in this case this is also a relation between a name and its bearer.

Among his published works, the one in which Frege argues in defense of the thesis that the reference of a sentence is its truth-value is the paper *On Sense and Reference* (Frege 1892, from now on *SR*), issued one year before the publication of the first volume of *The Basic Laws of Arithmetic* (Frege 1893, from now on *BLA*). The latter would be the main work of Frege's academic career, were it not for the inconsistency of the fateful Basic Law V. The purpose of the papers published in 1891 and 1892, *Function and Concept* (Frege 1891) and *SR*, was to present some adjustments in the formal system to be used in *BLA*. In particular, the main intent of *SR* was to establish the thesis that the reference (or better, the extension) of a sentence is its truth-value. This thesis, together with the sense/reference distinction for singular terms and Frege's theory of extensions (that ended up being inconsistent), yields an extensional logic for *BLA*. It is a fact, however, that Frege's argument in defense of truth-values as references presented in *SR* has some problems, and Frege himself seems in 1892 not to be completely convinced.

In fact, the above narrative has already been established as the standard interpretation of *SR*, but, in my view, there are some points of Frege's line of reasoning that are not yet fully explored. The aim of this paper is to show that Frege's argument may be reconstructed taking into account the failure of the notion of conceptual content, which in the *Begriffsschrift* (Frege 1879, from now on *BS*) played the role of semantic value of the expressions of his formal system, and also other passages where Frege presents the argument with a small but important difference, a letter to Russell of 1902 (Frege 1902)

and the text *Logic in Mathematics* (Frege 1914), published posthumously and dated 1914 by the editors of Frege (1979). I claim that the reconstructed argument, short and simple, is perfectly sound and convincing.

The text is structured as follows. In section 2 we remember some notions necessary for the discussion to be carried out, namely, the notion of semantic value in a formal system, and the distinction between intension and extension. In section 3, we discuss the problems of the notion of conceptual content presented by Frege in 1879 in *BS*. In section 4, Frege's argument in defense of truth-values as references is analyzed and reconstructed based not only in *SR* but also in Frege (1902) and Frege (1914).

2. Semantic values, extension, intension

In a formal system, the semantic value of a linguistic expression is a non-linguistic item associated to that linguistic expression. In a compositional semantics, the semantic value of a complex expression depends functionally on the semantic values of their constitutive parts and on the way they are combined. Let v be the semantic value of an expression A. When A is part of a more complex expression ($…A…$) the semantic value of ($…A…$) depends on v, and moreover, if ($…A…$) is a sentence, the semantic value of A may have a role to determine the truth-value of ($…A…$).

The semantic value associated to an expression A may be the extension or the intension of A. Roughly speaking, the intension of an expression A is the meaning of A, and the extension is what is designated by A. This is better explained by means of examples. The extensions of 'the author of *Nicomachean Ethics*' and 'the tutor of Alexander the Great' are the same, namely, Aristotle, but not their meanings. With respect to predicates, these distinction is well illustrated by the usual example of the predicates 'x is a human being' and 'x is a featherless biped'. Both have the same extension, because the set associated to each one is the same, but the intensions, or the way the elements of the set are picked out, or the meanings of the predicates, are not the same. The intension of a sentence is usually taken to be the proposition expressed, and the extension is its truth-value – of course, precisely the Frege's thesis discussed here.

We say that a logic **L** is intensional when it is concerned not only with the extensions but also with the intensions of the expressions of *L*, and that **L** is extensional when it is concerned only with the extensions of the expressions of *L*. In other words, a logic is extensional when what is relevant are not the meanings of its expressions, but only which entities are referred to by them.

With respect to singular terms and sentences, the intension/extension corresponds to the Fregean sense/reference distinction. With respect to predicates, Frege says in a posthumously published paper that the reference of a predicate is a concept, and that the sense is its mode of presentation (cf. Frege 1892b), but it is not clear what would be a mode of presentation of a concept. It is also worth noting that there is a point that may cause some confusion in the reading of *SR*. Although Frege distinguishes two semantic aspects associated with a linguistic expression, the reference and the sense, which correspond to the notions of extension and intension, the word 'reference' sometimes means 'extension' and sometimes means 'semantic value', for example, when Frege says that in indirect speech the reference of a sentence is the thought. In a more refined terminology, we would say that 'the reference of a sentence is its truth-value and the sense is the thought' means, 'the extension of a sentence is its truth-value and the intension is the thought', and 'in indirect speech, the reference of a sentence is the thought' means 'the semantic value of a sentence in an intensional contexts is the thought'.

3. The tensions in the notion of conceptual content in the *Begriffsschrift*

At the time of the *Begriffsschrift* (1879), Frege had not established yet the distinction between sense and reference. In that work, he talks about *conceptual content* and *judgeable content.* The latter is a special case of the former, since it qualifies the content of sentences, that is, contents that are true or false. I will talk here only about the more general notion, *conceptual contents*, introduced in the section 3 of *BS* and explained in terms of inferential role:

the contents of two judgments can differ in two ways: either the consequences derivable from the first, when it is combined with certain other judgments, always follow also from the second, when it is combined with these same judgments, [and conversely] or this is not the case. The two propositions 'The Greeks defeated the Persians at Plataea' and 'The Persians were defeated by the Greeks at Plataea' differ in the first way. (…)

I call that part of the content that is the *same* in both the *conceptual content*. (…)

[I]n a judgment I consider only that which influences its *possible consequences*. Everything necessary for a correct inference is expressed in full.

(Frege 1879 p. 12)

The conceptual content of a judgment, that is, a sentence, is what is relevant for inferences. Thus, we begin by saying that two sentences A and B have the same conceptual content if and only if they are intersubstitutable whilst preserving correctness of inference. However, by correctness of inference Frege did not have in mind what we understand by logical consequence. The former is not a sufficient nor a necessary condition for the latter. If all true arithmetical propositions are logical truths, as Frege held, then they will have the same conceptual content, but Frege certainly would not agree with this. Moreover, the example mentioned by Frege of a relation R and its inverse R^{-1} does not qualify as logical equivalence. In first order logic, $Rab \leftrightarrow R^{-1}ba$ is not a logical truth. There is no special notation for R^{-1}, the latter would be just a symbol different from R (note that $\forall x \forall y (Rxy \leftrightarrow R^{-1}yx)$ is not a valid formula of first order logic). So, logical equivalence cannot be a criterion of identity for conceptual contents.

Let us consider, for the sake of the argument, that $\Gamma \Vdash A$ means that through of one or more 'Fregean correct inferences' the sentence A may be obtained from the set Γ, that is, the symbol $\Vdash$ here means 'Fregean logical consequence'. So, from the passage quoted above, we get the following criterion for sameness of conceptual content of sentences:

(1) A and B have the same conceptual content *if and only if*

for any Γ and C: $\Gamma, A \Vdash C$ *iff* $\Gamma, B \Vdash C$.

that is tantamount to

(2) two sentences A and B have the same conceptual content *iff* $A \Vdash B$ and $B \Vdash A$.[2]

The natural way of extending this idea to singular terms is as follows:

(3) two singular terms a and b have the same conceptual content *if and only if*

$(\ldots a \ldots) \Vdash (\ldots b \ldots)$ and $(\ldots b \ldots) \Vdash (\ldots a \ldots)$.

However, we will see that so defined, this notion of conceptual content is not compatible with the notion of conceptual contents for singular terms.

In section 8 of *BS* Frege presents the sign of identity of content, $\equiv$. This sign is to be understood metalinguistically, that is, '$A \equiv B$' expresses a relation between the *signs* 'A' and 'B', namely, that 'A' and 'B' have the same content. He takes an example from geometry in which a point is determined in two different ways and introduces a distinction that is virtually the same as the later distinction between sense and reference with respect to singular terms.

> To each of these ways of determining the point there corresponds a particular name. Hence, the need for a sign for identity of content rests upon the following consideration: the same content can be completely determined in different ways; but that in a particular case *two ways of determining it* really yield the *same result* is the content of a *judgment*. Before this judgment can be made, two distinct names corresponding to the ways of determining the content, must be assigned to what these ways determine. (…)
> Now let
> $$\vdash (A \equiv B)$$

---

[2] in order to prove that (1) and (2) are equivalent, we need only that replacement property and monotonicity hold for $\Vdash$.

mean that *the sign A and the sign B have the same conceptual content, so that we can everywhere put B for A and conversely.*[3] (Frege 1879 p. 21)

The sign of identity of content appears also in section 24 of *BS*, where Frege introduces his notation for definitions:

$$\Vdash (A \equiv B)$$

Frege stipulates that expressions *A* and *B* have the same conceptual content.

> [A definition] differs from the judgments considered up to now in that it contains signs that have not been defined before; it itself gives the definition. It does not say "The right side of the equation has the same content as the left", but "It is to have the same content". (*Begriffsschrift* §24)

So, the symbol $\equiv$ is mentioned in these three places in 1879: §3, §8 and §24. Only in §8 it is an extensional operator.

The sign of identity of content, from the viewpoint of the later distinction between sense and reference, is a metalinguistic device that says that two expressions *A* and *B* may have different senses but the same reference. Based on the passage quoted above, we can say that

> (4) two singular terms *a* and *b* have the same content *if and only if* they pick up the same object.

Now consider the sentences

> (5) the author of *Nicomachean Ethic* is Greek

---

[3] The sign of identity of content, from the viewpoint of the later distinction between sense and reference, is nothing but a device that says that two expressions *A* and *B*, that may have different senses, have the same reference. Notice that this passage also justifies the proposition (3).

and

       (6) the tutor of Alexander the Great is Greek.

according to (4), (6) and (7) have the same conceptual content, but according to (3) they do not. Clearly, (6) and (7) are not intersubstitutable preserving correctness of inference because their consequences are not the same. The criteria presented in 1879, in sections 3 and 8 of *BS*, are incompatible. Therefore, conceptual contents cannot be the semantic values of the expressions of Frege's formal language.

       It is also worth noting that the Fregean notion of conceptual content yields a slingshot argument that collapses all true identity sentences of arithmetic. We need only to accept that

       (7) 'b is the successor of a' and 'a is the predecessor of b'

have the same conceptual content, what seems to be perfectly justified by the example given by Frege in *BS*. Now, all the sentences below have the same conceptual content:

[1] $0 =$ the predecessor of 1

[2] $1 =$ is the successor of 0   from [1] and (7)

[3] $1 =$ the predecessor of 2   from [2] and (4)

[4] $2 =$ the successor of 1   from [3] and (7)

and so on. By applying (4), further substitutions can be made, and so we get that *all true identity sentences $a \equiv b$ of arithmetic have the same conceptual content.*[4] Differently from Quine's, Davidson's and Church's slingshot (see Neale 2001), this argument does not result in the collapse of all conceptual contents, but it is enough to reject conceptual contents so defined.

4. A reconstruction of Frege's argument

In this section, the argument in defense of truth-values as references will be analyzed and reconstructed. But let me begin by taking a brief look at the thesis that concepts are functions whose values are truth-values, presented in 1891 in the article *Function and Concept* (Frege 1891). There, Frege extends the notion of function in order to allow functional expressions, that is, expressions that designate functions, with the symbols =, > and < (Frege 1891 p. 30). When Frege asks what would be the values of such functions, the thesis that the reference of a sentence is a truth-value appears in the form of the claim that concepts are functions whose values are truth-values. Notice that these two claims are virtually the same. The semantic value (i.e. reference) of the expression '$x^2 = 1$' is a function. The value of that function, say, for the argument *2*, will be the semantic value of the expression '$2^2 = 1$' (Ibidem). The claim that the values of the function designated by '$x^2 = 1$' for different arguments are truth-values and the claim that the references of sentences (for example, '$2^2 = 1$') are truth-values are one and the same.

The thesis that the reference of a sentence is its truth-value is presented and defended in *SR* (Frege 1892 pp. 62-64). Frege also argues in defense of this thesis in the posthumous *Logic in Mathematics* (Frege 1914 pp. 231-233) and in the aforementioned letter to Russell (Frege 1902 pp. 152-153). The most important difference between the arguments is the role of the principle of compositionality of reference,

(PCR) if Ref(*A*) = Ref(*B*), then Ref(…*A*…) = Ref(…*B*…).

In *SR, PCR* is a test applied by Frege after reaching the conclusion that the reference of a sentence is its truth-value. In both Frege (1902) and Frege (1914), it is clear that *PCR* is a premise essential to justify the last step of his argument.

---

[4] Although this fact has not been explicitly acknowledged by Frege, it constitutes one more piece of evidence for the thesis presented in *SR*, since what is common to all true identity sentences is precisely the fact that they are true.

Frege's line of reasoning proceeds through three steps: (I) Frege rules out the thought as the reference of sentences; (II) Frege concludes that sentences have reference; (III) Frege concludes that the reference of a sentence is its truth value. The critical step is the third.

4.1. Step I

In *SR*, after establishing the distinction between sense and reference for proper names, Frege asks if such a distinction should be extended to complete sentences. Frege assumes that a sentence expresses a thought and asks if the thought should be considered the reference of the sentence. The answer is negative. Thoughts as references of sentences are rejected based on *PCR* and a counterexample (Frege 1892, p. 62). Frege considers, as he had already done earlier in the paper, that the sentences

(8) The morning star is a body illuminated by the Sun

and

(9) The evening star is a body illuminated by the Sun

have different cognitive value, and so express different thoughts. But if the reference of a sentence were the thought expressed, the sentences (8) and (9) above should express the same thought, since the expressions 'the morning star' and 'the evening star' have the same reference.

4.2. Step II

The second step aims to answer whether or not sentences have reference. The answer will be positive (Frege 1892 pp. 62-63). From this part of the text of *SR*, together with Frege (1902 pp. 152-153) and Frege (1914 pp. 231-233), the following equivalences may be read:

The parts of a sentence have reference *if and only if*

the complete sentence has reference *if and only if*

the sentence has a truth value *if and only if*

the thought expressed is true or false.

However, the conclusion that sentences have reference may be obtained from a very short argument, as follows:

(10) If the parts of a sentence have reference, the complete sentence has reference.

(11) If we are interested in truth, (we require that) the parts of a sentence have reference

So, since in a scientific investigation we are interested in truth,

(12) the complete sentence has reference.

The premises (10) and (11) are justified by the following passages:

> The fact that we concern ourselves at all about the reference of a part of the sentence indicates that we generally recognize and expect a reference for the sentence itself. The thought loses value for us as soon as we recognize that the reference of one of its parts is missing. *On sense and reference* (Frege 1892 p. 63).

> What we talk about is the reference of words. We say something about the reference of the word 'Sirius' when we say: 'Sirius is bigger than the sun'. This is why in science it is of value to us to know that the words used have a reference. (…) The question first acquires an interest for us when we take a scientific attitude (…) Now it would be impossible to see why it was of value to us to know whether or not a word had a reference if the whole proposition did not have a reference and if this reference was of no value to us. *Letter to Russell* (Frege 1902 p. 152).

[it is essential] that the name 'Etna' should have a reference, for otherwise we should be lost in fiction. The latter of course is essential only if we wish to operate in the realm of science. (…)

If therefore we are concerned that the name 'Etna' should designate something, we shall also be concerned with the reference of the sentence as a whole. (…) That the name should designate something matters to us if and only if we are concerned with truth in the scientific sense. *Logic in Mathematics* (Frege 1914 p. 232).

4.3. Step III

After concluding that sentences have reference, in *SR*, Frege says

> We have seen that the reference of a sentence may always be sought, whenever the reference of its components is involved; and that this is the case when and only when we are inquiring after the truth value (Frege 1892 p. 63)

and concludes, somewhat reluctantly,

> We are therefore driven into accepting the *truth value* of a sentence as constituting its reference. (*Ibidem*)

Two paragraphs later Frege (1982 p. 64) applies *PCR* as a test in order to confirm, or make more plausible, his conclusion. However, both in Frege (1902) and Frege (1914), *PCR* is a premise of the argument.

> The reference of the proposition must be something which does not change when one sign is replaced by another with the same meaning but a different sense. What does not change in the process is the truth-value (Frege 1902 p. 152).

> The reference of a sentence must be something which remains the same, if one of the parts is replaced by something having the same reference. We return now to the sentence '(16 – 2) is a multiple of 7'. (…) what is not altered by replacing the sign '(16 – 2)' by the sign '(17 – 3)' is its truth value (Frege 1914 p. 232).

In my view, the best way to read Frege's argument is the following. Frege concluded that sentences have reference, and that the truth-value of a sentence is a plausible candidate, both because it shows itself as an alternative in the argument that concludes that sentences have reference, but also because it satisfies *PCR*, a necessary condition for being the reference. From this, Frege draws the conclusion that the reference of a sentence is its truth-value. So, the argument is

(13) The truth-value is a plausible candidate for being the reference of a sentence.

(14) If something is the reference of a sentence, it must satisfy *PCR*.

(15) The truth-value satisfies *PCR*,

therefore,

(16) The reference of a sentence is its truth-value.

The problem is that the above argument has not explicitly excluded something different from truth-values that could play the role of reference. Indeed, the argument from (14) and (15) to (16) is invalid – it is an instance of the well-known fallacy of affirmation of the consequent. Although Frege does not explicitly claim that there is no third option besides the truth-value and the thought for being the reference of a sentence, it is clear from the quotations from Frege (1914) and Frege (1902) that he was convinced that there was no third alternative.

4.4. The reconstructed argument

If we put the whole scenario in perspective, from 1879 (*BS*) to 1892 (*SR*), we get three options for being the semantic value of a sentence of Frege's formal system: (i) the conceptual content, (ii) the thought expressed, or (iii) the truth-value. The conceptual content has been excluded because of the problems discussed in the section 2 above. Thus, the argument may be reconstructed as follows:

(12) The complete sentence has reference.

(17) The truth-value satisfies *PCR*.

(18) The thought does not satisfy *PCR*.

(19) The reference of a sentence is either the truth-value or the thought.

So,

(20) The reference of a sentence is its truth-value.

Let us consider now a possible objection, namely, that there could still be a notion suited to be the reference but different from truth-values, thoughts and conceptual contents. Such a notion, let us call it Θ, would be either more fine-grained or less fine-grained than the thought, but it is clear that Θ is supposed to be more fine-grained then the truth-value. In both cases, however, Θ would still imply that the sentences 'b is the successor of a' and 'a is the predecessor of b' have the same reference, since the first can be defined from the second (or vice-versa), and it is not plausible to say that in a definition the *definiendum* and the *definiens* do not have the same reference. So, and again, all true identity sentences of Arithmetic would have the same reference. But a context where any true atomic sentence may be substituted by any other true atomic sentence is nothing but an extensional context, that is, a context in which the semantic value of a sentence is its truth-value.

5. Final remarks

The attempt to reconstruct Frege's line of reasoning is motivated by the feeling that it should be possible to extract from Frege's works a convincing argument in defense of truth-values as references, particularly because this is a very successful thesis and has naturally become the standard approach, as we learn from any book of elementary logic. The strategy here has been to investigate the problem from a viewpoint that considers the development of Frege's doctrines from 1879 to 1892. I have tried to show that the argument that

concludes that the reference of a sentence is its truth-value is not really in *SR*, but rather in the path that goes from *BS* to *BLA*.

We have seen that the main problem of Frege's argument in defense of truth-values as references is that it seems that he has not considered a possible third alternative besides the thought expressed and the truth-value, that would be something not extensional like truth-values. Actually, what shows that Frege did not have a third alternative are the problems of the notion of conceptual content. The latter, that in 1879 played the role of the semantic value for the expressions of Frege's formal language, is simultaneously intensional and extensional, with respect, respectively, to sentences and singular terms. Notice that such a mixture of intensionality with extensionality is directly responsible for the collapse that is the result of the slingshot argument.

The reason the thesis that truth-values are references of sentences seems implausible from the viewpoint of a theory of meaning is that the relation between a sentence and its truth-value is not like the relation between, say, 'Aristotle' and Aristotle. For the reader not familiar with the role of *SR* within Frege's work, it may seem that what is at stake in the first lines of *SR* is a contribution for a theory of meaning. Indeed, the discussion about sense and reference of singular terms is a contribution to a theory of meaning, but this is a secondary issue. But the supposed implausibility of truth-values as references, however, disappears when we realize that the central point of *SR* was not to present a theory of meaning, but rather to establish that the logic of *BLA,* to be published in the next year, is extensional. Frege was concerned in building an account of logical consequence designed to deal with mathematical propositions. So, from this point of view, which I claim to be the correct way to understand *SR*, Frege has *discovered* that such a logic must be extensional, and that the extension of a sentence is its truth-value.

6. References

Chateaubriand, O.: 2001. *Logical Forms – Part I.* Campinas: UNICAMP-Coleção CLE.

Dummett, M.: 1973. *Frege: Philosophy of Language.* Cambridge: Harvard University Press.

Frege, G.: 1879. '*Begriffsschrift*, a formula language, modeled upon that of arithmetic, for pure thought'. In van Heijnoort, J. (ed.) *From Frege to Gödel*, Lincoln: toExcel Press, 1999, pp. 1-82.

_____.: 1891. *Function and Concept.* In Frege 1960, pp. 21-41.

_____.: 1892. *On sense and reference.* In Frege 1960, pp. 56-78.

_____.: 1892b. *Comments on Sinn and Bedeutung.* In Frege 1997, pp. 172-180.

_____.: 1914. *Logic in Mathematics.* In Frege 1979 pp. 203-250.

_____.: 1902. *Letter to Russell.* In Frege 1980, pp. 152-154.

_____.: 1960. *Translations from the Philosophical Writings of Gottlob Frege.* Transl. Geach & Black. Oxford: Blackwell.

_____.: 1893. *The Basic Laws of Arithmetic: Exposition of the System, Volume I.* Transl. M. Furth. Los Angeles: University of California Press, 1964.

_____.: 1979. *Posthumous Writings*. Transl. Long & White. Oxford: Basil Blackwell.

 _____.: 1980. *Philosophical and Mathematical Correspondence*. Transl. H. Kaal. Chicago: University of Chicago Press, 1980.

_____.: 1997. *The Frege Reader.* (Ed. Beaney), Oxford: Blackwell.

Neale, S.: 2001. *Facing Facts.* Oxford: Oxford University Press.

Weiner, J.: 2001. 'Theory and elucidation: the end of the age of innocence' in: Floyd & Shieh (eds.), *Future pasts: the analytic tradition in 20th cent. Philosophy*. Oxford: OUP, 2001.

* * *