Peircean Pragmatic Truth and da Costa's Quasi-Truth

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Summary. In this paper we present a conception of the Peircean pragmatic truth and a formal definition of pragmatic truth, the *quasi-truth* - this concept, previously introduced by da Costa and collaborators, on trying to capture the meaning of the theories of pragmatist thinkers such as Peirce and James, is considered as the truth conception inherent to empirical theories and a generalization (for partial contexts) of Tarski's correspondence characterization of truth. By defining the mathematical concept of partial structure and by using a special semantical approach, we analyse a suitable logic that can be used as the underlying logic for theories whose truth conception is the quasi-truth. We delineate a Kripke model semantics for this logic and among some fundamental results we show that it is a kind of Jaśkowski discussive logic, a paraconsistent modal logic.

This conception of quasi-truth, the logic and the structures here presented can be useful for the analysis of model-based reasoning in empirical theories.

Key words: sign, model, truth, pragmatic truth, quasi-truth, partial structure, paraconsistent modal logic

1 Introduction

It is very difficult to develop any theoretical investigation without using the concept of truth. We cannot argue about a theory of truth without using this concept, because questioning a theory is to question its truthfulness, and accepting a theory is to accept it as true. We cannot leave out the concept of truth, as well as we can do with some other concepts.

According to [1]:

> ... we consider the classical concept of truth as a primitive concept. It is pressuposed in all our practical and theoretical activities. Philosophically, truth is a final concept, indefinable through other simpler concepts, if we used the term *definition* as a proposition that characterizes and explains, without *petitio principii*, a concept. The sentence

itself expressing, in strict sense, the definition of truth would have to be "true".

In [2], Lynch presents some connections between truth and other concepts: it is deeply connected to belief; it is also linked to knowledge; it is a central subject of logic in general; and it is also related to another mysterious concept, reality - in other words, to talk about truth is to talk of reality as it is.

We could investigate about two central subjects, concerning the property or underlying nature of truth:

1. Does truth even have a nature?
2. If so, what kind of nature?

The theories that try to answer this second question are frequently named *robust theories of truth*. Such theories consider that truth is an important property, that requests a substantial and complex explanation. Their defenders are interested in subjects, such as:

Either does the absolute truth exist, or is every truth somehow either subjective or relative?

Which type of relationship, if any, relates true propositions with the world?

Are all truths verifiable by sensitive experiences?

As all such subjects concern the objectivity of truth, according to [2], a fundamental subject for the robust theories is *realism*.

The *deflationary theories of truth* answer the first question negativily, leading to another debate: the deflacionists consider that the so-called problem of truth is in fact a pseudoproblem, truth does not constitute a property shared by all the propositions that we consider true. Therefore, the concept of truth should not be understood as expressing such property, but as playing another role, namely by considering that any explanation is unnecessary.

As in the robust theories, if we consider that truth is a property, then it is necessary to specify which things or what kind of things can present such property - the *truth-bearers*: in other words, which things may be either true or false. For [3], even in this case there is a lot of confusion: even if the philosophers agreed in identifying by a name the correct truth-bearer, the problem would hardly end, for they could disagree relatively to the nature of the things nominated by every one of those terms.

Among the robust theories of truth we have the correspondence, coherence and pragmatic theories of truth.

Correspondence truth is based on the idea that "truth is correspondence with reality", that is, a truth-bearer is true when the things in the world are as the truth-bearer says they are; if not, the truth-bearer is false. Besides, according to [2], in general it constitutes a realistic vision - if something is true, this does not depend on what everyone believes, truth depends on the

world and not on us. However, saying "truth is correspondence with reality" is nothing but a triviality. In order to establish a theory of correspondence truth, it is necessary to establish three of its aspects: which thing has the property of being true, i.e., what is the truth-bearer; the correspondence, i.e., what is the truth relationship; and the "reality" to which the truth-bearer corresponds.

Generically speaking, in a *coherence theory of truth*, a set of two or more beliefs is considered coherent if they "adjust" or "agree" among themselves (see [4]). Hence, the beliefs of an individual are true either if the set of his beliefs is coherent, or a belief is true if it is coherent with other beliefs in a system; on the contrary, they are false. Therefore, instead of being a correspondence relation between a truth-bearer and reality, in coherence theories the truth is a question of relationship between a truth-bearer and another truth-bearer. On account of that, this conception of truth is usually labelled as "epistemic".

The aim of this paper is to present a special conception and a formal definition of *pragmatic truth*, the *quasi-truth*. This concept, previously introduced by da Costa and collaborators, on trying to capture the meaning of the theories of pragmatist thinkers such as Peirce and James, is considered as the truth conception inherent to empirical theories and a generalization, for partial contexts, of Tarski's correspondence characterization of truth.

It is divided into two parts: in the first one, we briefly present an interpretation of Peirce's pragmatic concept of truth; we consider that, although it corresponds to a kind of partial truth, it is a kind of correspondence truth. Therefore, we need to explain what "reality" is, pointing the truth-bearer and the correspondence relationship: the sign, the correspondence relationship and the reality notion, as in Peirce, can be especially understood by addopting the scholastic realism, although Peirce's realism also contains elements of nominalism and idealism.

In the second part, by considering that models are signs, that mental action is an inferential process that uses Peirce's types of reasoning - *abduction*, *deduction* and *induction* -, and that scientific investigation has these same stages - abdutive, deductive and inductive -, we present da Costa's formal definition of pragmatic truth, based on models. By defining the mathematical concept of partial structure and by using a special semantical approach, we analyse an appropriate logic that can be used as the underlying logic for theories whose truth conception is da Costa's quasi-truth. We delineate a Kripke model semantics for this logic and among some fundamental results we show that it is a kind of paraconsistent modal Jaśkowski discussive logic.

2 Peircean Pragmatic Truth

2.1 Pragmatism

Pragmatism, the philosophical movement founded by Peirce, can be synthesized by the following passage, considered the "pragmatic maxim":

> Consider what effects, that might conceivably have practical bearings, we conceive the object of our conception to have. Then, our conception of those effects is the whole of our conception of the object. ([5] 5.402[1])

In this case, practical consequences are those effects of the conception that have influence in our practice, in our action. The meaning of an idea consists of its practical effects on the human experience. This way, if two ideas have the same practical consequences, then they have the same meaning, and ideas without practical consequences do not have any meaning. According to Peirce:

> Pragmatism, then, is a theory of logical analysis, or true definition; and its merits are greatest in its application to the highest metaphysical conceptions. ([5] 6.490)

In general, pragmatism can be understood as a method of either explaining ideas or determining meanings, a method that tries to take the techniques of experimental investigation to philosophical analysis. According to [6], pragmatism is characterized, in a quite wide way, by

i) An specific way of thinking - that approaches what is defended by the British empiricism, ..., concisely, the concretely observable is indispensable for the apprehension of meanings, as well as for the test of beliefs and ideas;
ii) An interpretation of life in evolutionistic terms, ..., concisely, the continuity and development are basic postulates for pragmatism;
iii) Adhesion to a naturalistic psychology - the spirit acts as specific functions of alive organisms, ...
iv) Acceptance of a scientific perspective in which experimentalism prevails.

2.2 Classification of Sciences

One of Peirce's main purposes is to delineate the underlying fundamental principles of the methods used in science, searching for constant elements in the different scientific methods.

Before studying any science, the philosophical thought should begin by a system of logic, whose first task must be establishing the most formal and universal categories of experience. Peirce concludes that there are only three formal elements or universal categories, omnipresent in every and any phenomenon: quality, relationship and representation, later named *firstness*, *secondness*, and *thirdness*. Peirce then distinguishes:

[1] References to Peirce's *Collected Papers* will be designated by [5], followed by the volume and paragraph numbers.

i) Three species of representations (or signs) - similarity (icon), index and symbol;
ii) A triad of conceivable sciences - formal grammar, logic and fomal rethoric;
iii) A general division of symbols, common to all those three sciences - terms, propositions and arguments;
iv) Three types of arguments: deduction (symbol), induction(index) and hypothesis or abduction (similarity or icon).

According to [7], there are three standpoints from which the categories have to be studied: *qualities*, *objects* and *mind*.

From the point of view of qualities (firstness), or ontological point of view, the categories appear as: quality or firstness - the being of positive qualitative possibility; reaction or secondness - action of the current fact; mediation or thirdness - the being of a law that will govern facts in the future.

From the point of view of objects (secondness), or of the existent, the categories appear as: quales - firstness facts; relationships - secondness facts; representations - signs or thirdness facts.

From the point of view of mind (thirdness), the categories appear as: feeling or immediate conscience - firstness signs; sensation of a fact - action sensation and reaction or secondness signs; conception - learning sense, mediation or thirdness signs.

As examples of those categories we have:

i) Firstness - indetermination, vagueness, possibility, originality, coolness, potentiality, quality, feeling;
ii) Secondness - certainty, final, object, correlative, reagent, being linked to, relationship notions, polarity, denial, matter, brute and blind force, compulsion, action-reaction, effort-resistance, here and now, opposition, effect, occurrence, fact, conflict, surprise, doubt, result;
iii) Thirdness - what is in development, generality, continuity, growth, mediation, infinite, intelligence, law, regularity, learning, habit, sign.

Peirce uses his categories as framework to his logical doctrine, as a basis for his classification of sciences. In decreasing order of abstraction:

1 Sciences of Discovery
  1.1 Mathematics
  1.2 Philosophy
    1.2.1 Phaneroscopy
    1.2.2 Normative sciences
      1.2.2.1 Aesthetics
      1.2.2.2 Ethics
      1.2.2.3 Logic or Semeiotics
        1.2.2.3.1 Pure or speculative grammar
        1.2.2.3.2 Critical logic
        1.2.2.3.3 Methodeutics or speculative rethoric
    1.2.3 Metaphysics

1.3 Special sciences
2 Sciences of revision
3 Practical sciences.

For Peirce mathematics is the science of exact conclusions, regarding hypothetical states of things, the only science that does not depend on the other ones. It is in philosophy that subjects regarding human experience are discussed and phaneroscopy has as its first task finding the most universal categories of experience.

Normative sciences are those that work either with the ends or the ideals that guide the feeling, the conduct and the human thought; they study the phenomena so that we can act on them and they on us.

Peirce attributes at least two senses to logic: the science of the necessary conditions to reach truth; and the science of the necessary laws of thought. He also considers logic as general semiotics, for treating the general conditions of signs as signs and the laws of thought evolution .

The aim of pure grammar, the first sub-division of logic, is to study all kinds of signs, their nature and meaning. The sign is mediation and belongs to thirdness.

Critical logic corresponds to what we nowadays know as mathematical-logic or logic and its aim is to investigate the conditions of truth of the logical inferences or arguments. However, in this case, Peirce introduces a new type of argument, the abduction - quasi-reasoning, discovery flash, responsible for the creation of hypotheses. So, critical logic was developed as an unified theory of abduction, induction and deduction.

Methodeutics studies the general conditions of the relationship between symbols (and other signs) and their interpretations. It is also a theoretical study whose aim is to examine the appropriate procedures to any investigation (see [7]).

2.3 Reality and Truth

According to the pragmatist maxim, the meaning of a conception of an object is constituted in the totality of its conceivable practical consequences; and its resultant action contains an element capable of moulding a future thought, accomplishing the rational purpose of the conception and having an intellectual element that permeates the deliberate conduct (see [8]). According to Peirce ([5] 7.361), thought is rational only when it refers to a possible future.

For [8], the core of Peirce's conceivable practical consequences is that a positive conception that a real object is supposed to have "should foresee the future course of the experience." And it is the action, or experience, that will reveal if there is a real conformity with the forecast: "correspondence between the theoretical forecast and the temporary course of the facts, the reinforcement of the conception is established in the form of a belief and, otherwise, as a doubt about its truthfulness."

For Peirce ([5] 5.372), the distinction between belief and doubt constitutes a practical difference. Beliefs guide our purposes and model our actions. A belief is an indication that some habits are more or less settled down, in such a way that they will determine our actions; they characterize a satisfactory and stable state that we either do not want to avoid, or do not want to shift to another belief; besides, different beliefs are distinguished by the different actions that they originate. On the other hand, doubt does not produce those effects, it constitutes a difficult and uncomfortable state that we struggle getting rid and passing to a state of belief; this uncomfortable state impels us to searching a stable state, a belief.

From the logical point of view, the investigation, the process of establishing stable beliefs, of establishing meanings, happens through three types of reasoning, according to Peirce:

> These three kinds of reasoning are Abduction, Induction, and Deduction. Deduction is the only necessary reasoning. It is the reasoning of mathematics. It starts from a hypothesis, the truth or falsity of which has nothing to do with the reasoning; and of course its conclusions are equally ideal. The ordinary use of the doctrine of chances is necessary reasoning, although it is reasoning concerning probabilities. Induction is the experimental testing of a theory. The justification of it is that, although the conclusion at any stage of the investigation may be more or less erroneous, yet the further application of the same method must correct the error. The only thing that induction accomplishes is to determine the value of a quantity. It sets out with a theory and it measures the degree of concordance of that theory with fact. It never can originate any idea whatever. No more can deduction. All the ideas of science come to it by the way of Abduction. Abduction consists in studying facts and devising a theory to explain them. Its only justification is that if we are ever to understand things at all, it must be in that way ([5] 5.145).

Induction, from a given theory, looks for facts that prove its truthfulness; on the other hand, abduction, from facts, looks for a theory, that is, from the observed experience it constructs the concepts - abduction constitutes the creative reasoning of the ideas of science, of the hypotheses and in a general way of every creation.

In abduction the consideration of the facts suggests the hypotheses. In induction the study of the hypotheses suggests the experiments that bring to light the very facts to which the hypotheses had pointed ([5] 7.218).

In *Questions Concerning Certain Faculties Claimed for Man* (1868) ([5] 5.213-263), *Some Consequences of Four Incapacities* (1868) ([5] 5.264-317) and *Grounds of Validity of the Laws of Logic: Further Consequences of Four Incapacities* (1869) ([5] 5.318-357), Peirce criticizes *cartesianism*, based mainly on the concept of intuition. One of his conclusions is that mental action is an inferencial process and that thought only works through signs. According to

[9], such rejection to the cartesian conception is in the basis of the Peircean theories of mental action, signs, cognition, scientific investigation, methods, human insight and discovery, and of pragmatism.

In that way, we consider that the Peircean conception of truth is strongly related to such concepts. We also consider the Peircean truth as a kind of correspondence truth, in other words, a relation among truth-bearers (the signs) and reality. However, such relationship does not express an absolute truth, but a partial truth. This partiality depends on Peirce's sign definition and reality notion.

In spite of Peirce's realism containing elements of the scholastic realism, nominalism and idealism - called "realicism" by [10] -, sometimes Peirce identifies himself as a "scholastic realist", mainly (but not totally) furthering Duns Scotus' realism. Peirce retakes one of the great controversies of Medium Age, the problem of the universals - that he prefers to denominate the "generals", in defense of his realism, against nominalism.

For Peirce ([5] 8.12), the *real* "is that which is not whatever we happen to think it, but is unaffected by what we may think of it."

The *scholastic* or *moderate realism* is placed between two extremes: the *platonic realism* - the universal do exists -, and *nominalism* - the universal is not real. The scholastic realism recognizes that, although the existence only of the individual, the universal can be real.

Nominalism considers that only the individuals exist, only these are real. The universal is not real, because it does not exist, it is mere words as many concepts, and always dependent on the mind.

Peirce considers that the adoption of nominalism is a mistake, because nominalism considers only one way of being, the being of a thing or an individual fact, the existence. For Peirce, the laws and every type of regularity govern future facts - that do not exist -, and they are real; but nominalism cannot explain what is a law, since it recognizes only the current existence. However, realism claims that the existence of individual things is not the only way of being.

Although the universals being mind dependent concepts, they can be considered real. From the epistemological point of view it seems that we acquire knowledge of the world through a generalization process. But, in that way, what the intellect knows on a sensorial object is not what is individual, but only its general characteristics. However, this does not impede that we know the world as really it is ([10]).

For Peirce ([5] 8.12), the real, the independent thing of what someone thinks of it, is not out of the mind, because the intellect attributes an important component to the concept of thing. The real is independent of my thoughts, as well as yours, but it is not independent of thought in general. Thought and human opinion contain an arbitrary, accidental element that depends on the limitations, circumstances and the individual power. But human opinion, in the long run, tends to a defined form, that is the truth. Any human being that has enough information and thinks sufficiently on any subject, will

reach a certain defined conclusion, that will be the same one of any other mind under sufficiently favorable circumstances. The arbitrary will or other individual peculiarities of a sufficiently large number of minds can indefinitely postpone the general agreement on a subject, but they cannot affect the character that such opinion will have when they are reached. And the final opinion is the truth, because this opinion is independent, certainly not independent of thought in general, but of everything that is arbitrary and individual in the thought. Everything that will be thought as existing in the final opinion is real, and nothing else. Therefore the scientific method is the best one to acquiring stable beliefs. For although being fallible and partial, such truths will be the fruit of the agreement of a community of investigators and we can have knowledge of the world because human thought, in general, tends to truth.

3 Da Costa's Quasi-Truth

Tarski, when introduced his formal definition of correspondence truth, the "semantical conception of truth for formalized languages", sought "to capture" the existing intentions in "aristotelian classic conception of truth" (see [11, 12, 13]). Similarly, [14] tried to represent the "intentions" of the theories of truth of pragmatists such as Peirce and James (cf. [15]): loosely speaking, they say that a sentence is pragmaticaly true if, in a certain context, "it saves the appearances", i.e., if in that context it goes correspondentialy true.

Mikenberg, da Costa and Chuaqui observe that formal definitions are, at least in principle, neutral, or at least as neutral as the mathematical formulations in which they are represented. From the formalism of set theory, they introduce a formal version of the notion of pragmatic truth, conveniently adapting Tarski's definition.

According to [16], the intentions of the pragmatist vision of truth represent an emphasis on:

i) The nature of the agreement between "imperfect" or "abstract" description and reality;
ii) The empirical consequences of such descriptions, understood as "agreement" with reality, in the classic correspondence sense;
iii) The "complete" or "absolute" truth, again understood in the classic correspondence sense, as (ideal) end of every investigation.

From the naturalistic change in the philosophy of science, the nature and importance of scientific practice have been reevaluated. However, a problem that appears is that no construction of reasoning can accomodate the vagueness and complexities of such practice. According to da Costa and French, a unitary treatment can be built, that incorporates and focuses two fundamental aspects of the epistemic practice in general, concerning the nature

of representation used in the scientific reasoning and the epistemic attitudes adopted relatively to it, and the methodology.

The representations are, basically, conceptually incomplete and unfinished, the adopted general attitude is falibilist. The representations used in the scientific practice are not seen as true in the correspondence sense, but as partially true, approximately true or as containing some truth element - the development of a formal concept of pragmatic truth can eliminate the deficiencies of the attempts of formally capturing such notions.

The definition of quasi-truth offers a way of accomodating the incompleteness inherent to scientific representations, with the introduction of the notion of partial structure, in the semantic approach of theories through the introduction of an adequate model theory.

Tarski's definition is extended to the quasi-truth definition:

i) The notion of structure is extended, by introducing the notion of "partial structure";
ii) The notion of "quasi-truth" is introduced, being a generalization of Tarski's characterization of truth for partial contexts.

In general, when we investigate a certain domain of knowledge, we don't know everything about it, in other words, our information is incomplete or partial. Therefore, we cannot be sure that a particular theory on that domain is true, but we can say that, as much as our information allow us, such theory can be true, i.e., it is quasi-true. According to [17]:

i) When a certain domain Δ of knowledge is investigated, we submit it to a conceptual scheme, in order to systematize and organize the information about it;
ii) That domain is "acted" by a set D of objects, and is studied *via* the analysis of the relations among its elements.
iii) Given a relation R on D, as it frequently happens in scientific contexts, "we do not know" if all the objects of D are related by R;
iv) Therefore, we say that our information about Δ is "incomplete" or "partial."

The introduction of the notions of partial relation and partial structure makes possible to formally accomodate that incompletness and to represent the information about the investigation domain.

Definition 1. *Let D be a non-empty set. A n-ary partial relation R on D is a triple $\langle R_1, R_2, R_3\rangle$, where $R_i \cap R_j = \emptyset$, for $i \neq j$, $i, j \in \{1, 2, 3\}$ and $R_1 \cup R_2 \cup R_3 = D^n$, such that:*

i) R_1 is the set of n-tuples that we know that belong to R;
ii) R_2 is the set of n-tuples that we know that do not belong to R;
iii) R_3 is the set of n-tuples that we don't know whether they belong to R or not.

We observe that if $R_3 = \emptyset$, R is an usual n-ary relation, that can be identified with R_1.

Definition 2. *A partial structure A is an ordered pair $\langle D, R_i\rangle_{i\in I}$, where:*

i) D is a non-empty set;
ii) $(R_i)_{i\in I}$ is a family of i-ary partial relations on D.

As in the notion of pragmatic truth correspondence truth is involved, also in the definition of quasi-truth Tarski's characterization of truth is involved. For Tarski, a sentence of a first-order language L is true or false, only relatively to a certain interpretation in a given structure: similarly, a sentence can be quasi-true or quasi-false, only relatively to an appropriate type of structure. But, as in Tarski's characterization only total structures are used (in which the relations are usual, non-partial), intermediate notions of structures are here defined, to establish a relationship between partial and total structures.

Definition 3. *A simple pragmatic structure (sps) for a first-order language L is a structure $A = \langle D, R_k, \wp\rangle_{k\in I}$, where:*

i) D is a non-empty set, the universe of A;
ii) R_k is a k-ary family of partial relations on D, for all $k \in I$ (R_k may be empty, for some k);
iii) $\wp$ is a set of sentences of L.

A simple pragmatic structure is a partial structure with a third component: a set of sentences $\wp$ of L, either accepted as true or that are true according to the correspondence theory; these sentences can express either true statements, empirically decidable, or general sentences expressing either laws or theories accepted as true.

Given a simple pragmatic structure, it can be extended to a total structure.

Definition 4. *Let L be a first-order language, $A = \langle D, R_k, \wp\rangle_{k\in I}$ a sps and S a total structure, where L is interpreted . S is an A-normal structure if the following properties hold:*

i) The universe of S is D;
ii) The (total) relations of S extend the correspondent partial relations of A;
iii) If c is an individual constant of L, then c is interpreted in A and S by the same element;
iv) If $\alpha \in \wp$, then S satisfies α, i.e., every sentence of $\wp$ is valid in the structure S, what is denoted by $S \models \alpha$.

Definition 5. *Let L be a language, A a sps and S an A-normal structure. A sentence α of L is quasi-true in the sps A, relatively to S, if α is true in S according to Tarski's definition of truth. Otherwise, α is quasi-false.*

In other words, if α is quasi-true in A then all the logical consequences of α, or α plus the primary declarations $\wp$, should be compatible with any true primary declaration. Hence, α is such that everything happens in the domain of knowledge under investigation Δ as if α was true.

4 A Logic for Quasi-Truth

In [17], Chapter 4, it is analysed a logical system that can serve as the underlying logic to theories that have the quasi-truth as their truth conception. In general, this logic can be used as a deductive logic of science.

In order to build this logic of pragmatic truth, from a first-order language L and a given sps A that interprets L, we consider its A-normal structures as "worlds" of a Kripke structure for the *first-order with equality modal system* S5Q$^=$ - a S5Q$^=$ model. That is, from the universe of a sps A for L, we have several structures (total) in which L can be interpreted, such that any total structure is accessible to the other.

In L (and in A), the possibility operator $\Diamond$ corresponds to the quasi-truth notion (pragmatic truth) and the necessity operator (∇) to the quasi-validity notion (pragmatic validity). In order to formalize these two notions, we deal with two logical systems, S5Q$^=$ and QT.

The pragmatically valid formulas are the formulas α such that $\nabla\alpha$ is a theorem of S5Q$^=$. Among these, there are formulas $\nabla\Diamond\alpha$ such that $\Diamond\alpha$ is a theorem of S5Q$^=$. We name the first class of formulas *strict-pragmatically valid*, or simply *strictly valid* formulas (the theorems of S5Q$^=$); the second class is named *pragmatically valid* formulas, that are the theorems of da Costa's *system QT - a paraconsistent modal system* associated to S5Q$^=$, a kind of Jaśkowski's discussive logic, a logic for quasi-truth ([1]).

The language L of QT is the language of S5Q$^=$. The axioms and inference rules are the following.

Axiom 1. If α is an instance of a classical propositional tautology, then $\nabla\forall\forall\alpha$ is a QT-axiom.
Axiom 2. $\nabla\forall\forall(\nabla(\alpha \to \beta) \to (\nabla\alpha \to \nabla\beta))$
Axiom 3. $\nabla\forall\forall(\nabla\alpha \to \alpha)$
Axiom 4. $\nabla\forall\forall(\Diamond\alpha \to \nabla\Diamond\alpha)$
Axiom 5. $\nabla\forall\forall(\forall x\alpha(x) \to \alpha(t))$
Axiom 6. $\nabla\forall\forall(x = x)$
Axiom 7. $\nabla\forall\forall(x = y \to (\alpha(x) \leftrightarrow \alpha(y)))$
Axiom 8. In any formula, empty quantifications can be either introduced or suppressed.

Rule 1. $\vdash \nabla\forall\forall\alpha, \ \vdash \nabla\forall\forall(\alpha \to \beta) / \vdash \nabla\forall\forall\beta$
Rule 2. $\vdash \nabla\forall\forall\alpha / \vdash \alpha$
Rule 3. $\vdash \nabla\forall\forall\alpha / \vdash \nabla\forall\forall\nabla\alpha$
Rule 4. $\vdash \Diamond\forall\forall\alpha / \vdash \alpha$
Rule 5. $\vdash \nabla\forall\forall(\alpha \to \beta(x)) / \vdash \nabla\forall\forall(\alpha \to \forall x\beta(x))$

Hifume presents specific definitions and proves fundamental results of QT.

Definition 6. *A QT-model is a S5Q$^=$-model.*

Definition 7. *In QT, a well formed formula (wff) α is a semantical-pragmatic consequence of a set Γ of wff of L, what is denoted by $\Gamma \models^{p}_{QT} \alpha$, if, and only if, there are formulas $\gamma_1, \gamma_2, ..., \gamma_n$ in Γ such that $\models_{QT} \Diamond\gamma_1 \wedge \Diamond\gamma_2 \wedge ... \wedge \Diamond\gamma_n \rightarrow \Diamond\alpha$.*

Theorem 1. *For every wff α of L, α is a theorem of QT if, and only if, $\Diamond\forall\forall\alpha$ is a theorem of S5Q$^{=}$:*

$$\vdash_{QT} \alpha \quad \Leftrightarrow \quad \vdash_{S5Q^{=}} \Diamond\forall\forall\alpha.$$

Theorem 2. *If a wff α is a theorem of S5Q$^{=}$, then it is a theorem of QT:*

$$\vdash_{S5Q^{=}} \alpha \quad \Rightarrow \quad \vdash_{QT} \alpha.$$

The converse of Theorem 2 is false, i.e., QT is "stronger" than S5Q$^{=}$. And, in QT, *Modus Ponens* does not hold, relatively to material implication.

Definition 8. *In QT, a formula α is a syntactic-pragmatic consequence of a set Γ of wffs, what is denoted by $\Gamma \vdash^{p}_{QT} \alpha$, if there are $\gamma_1, \gamma_2, ..., \gamma_n$ in Γ such that $(\Diamond\gamma_1 \wedge \Diamond\gamma_2 \wedge ... \wedge \Diamond\gamma_n) \rightarrow \Diamond\alpha$ is a theorem in QT.*

Definition 9. *A pragmatic theory whose underlying logic is QT, is a non-empty set Σ of sentences such that, if $\gamma_1, \gamma_2, ..., \gamma_n$ are in Σ and $\{\gamma_1, \gamma_2, ..., \gamma_n\} \vdash^{p}_{QT} \alpha$, then α is also in Σ.*

Theorem 3. *If Σ is a pragmatic theory and α is a theorem in QT, then $\alpha \in \Sigma$.*

Definition 10. *Let E be the set of all sentences of QT and Σ a pragmatic theory. Σ is trivial, or overcomplete, if Σ=E; otherwise, Σ is non-trivial. The theory Σ is inconsistent (contradictory), if there exists at least a sentence α such that $\alpha \in \Sigma$ and $\neg\alpha \in \Sigma$, where $\neg$ is the negation symbol of QT; otherwise, Σ is consistent (non-contradictory).*

Theorem 4. *There are inconsistent, but non-trivial, pragmatic theories.*

According to [18], a logic is *paraconsistent* if it can be used as the underlying logic for inconsistent but non-trivial theories, named *paraconsistent theories*. In this sense, QT is a paraconsistent logic.

Definition 11. *Let α and β be wffs of L:*

1. *Pragmatic implication $\rightarrow_p$: $\alpha \rightarrow_p \beta =_{df} \Diamond\alpha \rightarrow \beta$*
2. *Pragmatic conjunction $\wedge_p$: $\alpha \wedge_p \beta =_{df} \Diamond\alpha \wedge \beta$.*

Remark 1. Let α, β be wffs of L. In general, the *Pseudo-Scotus* Principle does not hold in QT, relatively to the pragmatic implication:

$$\not\models_{QT} \alpha \rightarrow_p (\neg\alpha \rightarrow_p \beta).$$

Hence, according to the literature, QT is paraconsistent *lato senso* relatively to the pragmatic implication $\rightarrow_p$.

Proposition 1. *For every wff α and β in QT, Modus Ponens Rule holds, relatively to the pragmatic implication.*

Theorem 5 (Pragmatic Deduction Theorem). *For every wff α and β, β is a syntactic-pragmatic consequence of α if, and only if, the pragmatic implication $\alpha \rightarrow_p \beta$ is a theorem in QT:*

$$\alpha \vdash^p_{QT} \beta \quad \Leftrightarrow \quad \vdash_{QT} \alpha \rightarrow_p \beta.$$

Theorem 6 (Completeness). *The wff α is pragmatically valid if, and only if, α is a theorem in QT:*

$$\models_{QT} \alpha \quad \Leftrightarrow \quad \vdash_{QT} \alpha.$$

Theorem 7 (Pragmatic Completeness). *Let Γ be a set of wff and α a wff of L. α is a semantical-pragmatic consequence of Γ if, and only if, α is a syntactic-pragmatic consequence of Γ in QT:*

$$\Gamma \models^p_{QT} \alpha \quad \Leftrightarrow \quad \Gamma \vdash^p_{QT} \alpha.$$

5 Final Considerations

Axiomatization constitutes a formal method of specifying the content of a theory. Given a formal language, from a set of axioms, rules of inference and definitions, the content of the theory can be deductively derived as its theorems. The theory is then identified to the set of axioms and their deductive consequences.

According to da Costa, the axiomatic method leads to a economy of thought : when we study an abstract axiomatics, we are simultaneously treating several theories - all that are framed in the considered axiomatics. By the axiomatic method we can also investigate problems such as the equivalence of theories or the independence of axioms. It also constitutes adequate tools for mathematical work and research. And, in general, the deductive disciplines are based on the norms of the axiomatic method.

In mathematics, from Bourbaki's structural approach, the axiomatic method reaches a high level of precision and development: axiomatizing a mathematical theory consists in defining a type of structure, based on a set theory; a structure consists of a non-empty set and relationships among its elements, satisfying certain conditions imposed by a set of axioms. But such a formalization is essentially syntactic. A type of structure is seen as simply constituting a formal theory, built as a collection of symbols subjected to certain metamathematical rules. However, although such method has suffered critics, according

to [19] the axiomatization continues being considered as an important component of the philosophy of science, for its role in the clarification of the basic concepts of a theory; its aid for the comparison of theories; the way it can allow the use of mathematical techniques; and for its usefulness in solving certain philosophical disputes.

However, in the philosophy of science, an alternative method to the axiomatization of theories - essentially syntactic - is the semantical approach of theories or the model theory. According to [16], the introduction of model theory by Tarski, by formalizing the notion of correspondence truth in terms of "the sentence α is true in a structure S", suggests that other semantical notions could be defined in a similar way. In this approach, the semantical tally of scientific theories should be seen, not as sets of sentences axiomatized in some appropriate formal language, but as classes of models.

In spite of the great development reached by the formal sciences, their representations are still, essentially, deductive, atomist and they use Tarski's conception of truth. Based on Peirce's semeiotic conception of knowledge, on his sign definition, his conceptions of truth and reality, and on his metaphysics, we claim that a more coherent way of representing knowledge and cognitive processes can be obtained - models and systems are kinds of signs, that work with several other kinds of signs through abduction, deduction and induction.

The notions of partial structure and quasi-truth presented in this paper have other important applications in the theory of science, as for instance in the theoretic unification in science ([16]), in pragmatic probability ([20]), in the logic of induction, in inconsistent beliefs, in the realism-empiricism debate.

In future works, we intend to pursuit our research by introducing and developing a theory of non-classical models, from some Peircean ideas and da Costa's quasi-truth definition.

References

1. da Costa, N.: O Conhecimento Científico. 2nd. edn. Discurso Editorial, São Paulo (1999)
2. Lynch, M.: The Nature of Truth: classic and contemporary perspectives. The MIT Press, Cambridge (1992)
3. Kirkham, R.: Theories of Truth: a critical introduction. The MIT Press, Cambridge (1992)
4. Kirkham, R.: Coherence theory of truth. In: Routledge Encyclopedia of Philosophy. Volume 9., London/New York, Routledge (1998) 470–2
5. Peirce, C. In C. Hartshorne, P.W., Burks, A.W., eds.: The Collected Papers of Charles S. Peirce. Harvard University Press, Cambridge (1931-1966) 8 vols.
6. Hegenberg, L., da Mota, O.: Introdução. In: Semiótica e Filosofia: textos escolhidos de Charles Sanders Peirce, São Paulo, Cultrix (1972)
7. Santaella, L.: Matrizes da Linguagem e Pensamento: Sonora, Visual, Verbal. Iluminuras/FAPESP, São Paulo (2001)
8. Ibri, I.: Kósmos Noētós: a arquitetura metafísica de Charles S. Peirce. Perspectiva/Hólon, São Paulo (1992)

9. Santaella, L.: O Método Anticartesiano de C.S. Peirce. Editora Unesp, São Paulo (2004)
10. Mayorga, R.: On Universals: the scholastic realism of John Duns Scotus and Charles Peirce. University of Miami, Miami (2002)
11. Tarski, A.: The concept of truth in formalized languages. In Corcoran, J., ed.: Logic, Semantics, Metamathematics. Hackett, Indianopolis (1933) 152–278
12. Tarski, A.: The semantic conception of truth. Philosophy and Phenomenological Research **4** (1944) 13–47
13. Tarski, A.: Logic, Semantics, Metamathematics. 2nd. edn. Hackett, Indianopolis (1983)
14. Mikenberg, I., da Costa, N., Chuaqui, R.: Pragmatic truth and approximation to truth. The Journal of Symbolic Logic **51** (1986) 201–21
15. James, W.: Pragmatismo. In: Os Pensadores, São Paulo, Nova Cultural (1989) Translated into Portuguese by J.C. da Silva.
16. da Costa, N., French, S.: Science and Partial Truth: a unitary approach to models and scientific reasoning. Oxford University Press, Oxford (2003)
17. Hifume, C.: A Pragmatic Theory of Truth: Newton C.A. da Costa's quasi-truth. State University of Campinas, Campinas (2003)
18. D'Ottaviano, I.: On the development of paraconsistent logic and da costa's work. The Journal of Non-Classical Logic **7**(1/2) (1990) 9–72
19. da Costa, N., French, S.: The model-theoretical approach in the philosophy of science. Philosophy of Science **57** (1990) 248–65
20. da Costa, N.: Pragmatic probability. Erkenntnis **25** (1986) 141–62