Zbl 0685.03001

Epstein, Richard L.; Carnielli, Walter A. Computability. Computable functions, logic, and the foundations of mathematics. (English) [B] Wadsworth & Brooks-Cole Mathematics Series. Pacific Grove, CA: Wadsworth & Brooks-Cole Advanced Books & Software. xvii, 297 p. (1989). ISBN 0-534-10356-1.

In this introduction to logic, computability and the foundations of mathematics, which does not call for any serious knowledge of mathematics on the part of the reader, emphasis is laid upon the philosophical problems that guided and furthered the development of the subject. The book is well-organized and it contains extensive quotes from many seminal papers. It is divided into four parts: Parts II and III form the mathematical core of the book, whereas parts I and IV are mainly philosophical. Part I is introductory; starting from the paradoxes, it gives a short treatment of the notions of number, function, proof and the infinite, and it concludes with a 14-page quotation from *D. Hilbert's* paper "On the infinite". Part II develops the theory of computability. Turing machines are introduced and the class of recursive functions is also characterized in Kleene's way by means of the μ-operator. There are asides on the Grzegorczyk hierarchy, and on multiple recursion. The heuristic evidence for Church's Thesis is referred to as The Most Amazing Fact. Part III expounds Gödel's incompleteness theorems; on the way, the reader is acquainted with propositional logic and first-order-logic. Part IV offers a discussion of various arguments concerning Church's Thesis and a survey of several positions within so- called constructive mathematics: intuitionism, recursive mathematics, Bishop-style constructivism, and finitism.

The book contains a good many exercises, and also a lot of questions inviting the reader to philosophical argument. It is a pleasure to read it, it has good references, also to the very recent literature, and it will undoubtedly prove its merits when used in class-room or as secondary reading. There must be a Teacher's Manual, but I did not see it.

The book invites comparison to its famous predecessor by *G.S. Boolos* and *R.C. Jeffrey* [Computability and logic (1974; Zbl 0298.02003)]. Although the latter book addresses itself to roughly the same readership as the book under review, there are some differences. Boolos and Jeffrey assume that the reader already knows some logic, and they also offer several more advanced theorems from logic which do not occur in the present book, such as the compactness theorem, the Skolem-Löwenheim theorem, the Craig interpolation lemma, Ramsey's theorem and the modal treatment of provability. One has to admit that the present book is on a more elementary level. The authors deserve praise for the way in which they reach their more modest aims, however: I expect many readers to enjoy the copious references to the historical and philosophical incentives to the problems of mathematical logic. The authors also succeeded in giving their book a unifying theme: the formulation of Hilbert's programme, and its demise by the hands of Gödel. I was gladdened by the authors' ample discussion of constructivist positions in the last chapter of the book: this subject is conspicuously absent in many introductions to mathematical logic. (I spotted one mistake, on page 232, line 4 from below, where the authors, quoting Heyting, explain the definition of P_n given on line 5 from below; it should read: $\{$ x: x$<$ n and $\phi_x(x)\downarrow\}$ and not: $\{$ x: $\phi_x(x)\downarrow$ in $\leq n$ steps$\}$. The latter set is recursive, intuitionistically as well as classically.) [W.Veldman]