Logics of Deontic Inconsistency

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ABSTRACT: The *Logics of Formal Inconsistency* (LFIs) are paraconsistent logics which internalize the notions of *consistency* and *inconsistency* by means of connectives. Based on that idea, in this paper we propose two deontic systems in which contradictory obligations are allowed, without trivializing the system. Thus, from conflicting obligations $O\varphi$ and $O\neg\varphi$ contained in (or derived from) an information set, it can be derived that the sentence φ is deontically inconsistent. This avoids the logic collapse, and, on the other hand, this allows to "repair" or to refine the given information set. This approach can be used, for instance, for analyzing paradoxes based on contrary-to-duty obligations.
KEYWORDS: Logics of formal inconsistency, deontic logic, contrary-to-duty obligations.

1. Introduction

Inconsistencies occur in several contexts, for instance databases and logical paradoxes. Within a classical framework, inconsistencies are identified with contradictions. Paraconsistent logics and, in particular, the so-called *Logics of Formal Inconsistency*, LFIs (see CARNIELLI, MARCOS, 2002 and CARNIELLI, CONIGLIO, MARCOS, 2007), deal with contradictions without trivializing the logic system. Thus, contradictions are not necessarily inconsistent (that is, without models or, equivalently, logically trivial or *senseless*), and an additional hypothesis of *consistency* of the contradictory formula is necessary, in order to equate contradictoriness with triviality. The consistency of a formula φ is made explicit by means of an unary *consistency* operator, o, such that oφ expresses that φ is 'consistent' (or 'safe', or 'well-behaved', or 'conclusive').

In a dual perspective, the *inconsistency* of a sentence φ can be expressed by a sentence of the form $\bullet\varphi$, where $\bullet$ is an *inconsistency operator*. In most LFIs both operators are interdefinable as expected:

$$\bullet\varphi \equiv \neg\circ\varphi \quad \text{and} \quad \circ\varphi \equiv \neg\bullet\varphi.$$

Here, we use the standard symbol "$\neg$" to denote the connective of (paraconsistent) negation within a given LFI. By its turn, the symbol "$\equiv$" denotes the logical equivalence of two given sentences within a LFI.

The approach of LFIs to paraconsistency is also useful for *detecting* inconsistencies without falling in a logic collapse. This feature of LFIs can be applied, for instance, to remove inconsistencies in databases: a suitable logic tolerating contradictions and marking (with an inconsistency operator $\bullet$) such contradictory sentences could help to "clean up" and to refine the database (see, for instance, CARNIELLI, MARCOS, DE AMO, 2000).

Based on this idea, we propose two deontic systems, called *Logics of Deontic Inconsistency*, which are deontic extensions of LFIs and then the negation operator in these logics is paraconsistent. Thus, from contradictory obligations $O\varphi$ and $O\neg\varphi$ it can be inferred that φ is *deontically inconsistent* without trivializing the logic. Here, O denotes the "obligation" connective such that the sentence $O\varphi$ represents that "φ is obligatory". The notion of *deontic inconsistency* is expressed in our framework by a sentence of the form $\otimes\varphi$, where $\otimes$ is a new unary connective. Additionally, an adequate Kripke semantics is presented for these systems.

By using logics of deontic inconsistency, deontic paradoxes are not solved but, instead, the information set containing such conflictive sentences (for instance, a database) can be "repaired" and better understood. In fact, the derivation of sentences of the form $\otimes\varphi$ from a logically trivial (w.r.t. *Standard Deontic Logic* **SDL**) set of sentences helps to detect which sentences are involved in a deontic conflict. Being so, a decision can be taken (for instance, to remove them from the database or to modify them) in order to overcome the conflict. Additionally, this kind of logics could be useful to analyze, for instance, moral dilemmas.

2. Logics of Formal Inconsistency

From now on, the symbols $\neg$, $\wedge$, $\vee$, $\Rightarrow$ and $\circ$ will be used to represent the connectives of negation, conjunction, disjunction, implication and consistency, respectively. On the other hand, the symbol $\vdash$ will be used to represent the consequence relation of a given logic. To be strict, each logic under consideration should require the use of a different symbol for its consequence relation; however, we will allow the use of the same symbol $\vdash$ to denote the consequence relation of different logics, in order to simplify the notation.

As it is well known, a basic principle of classical logic is the so-called *Principle of Non-Contradiction*, which states that

$$\neg(\varphi \wedge \neg\varphi)$$

is a theorem, for any formula φ. This principle can be seen (within the framework of classical logic) as a direct consequence of the *Ex Contradictione Sequitur Quodlibet* property:

$$\varphi, \neg\varphi \vdash \psi$$

for every φ and ψ. The latter can be axiomatized through the schema

$$\varphi \Rightarrow (\neg\varphi \Rightarrow \psi).$$

In general, the Logics of Formal Inconsistency LFIs) consider a weaker version of this axiom, say

(bc) $\circ\varphi \Rightarrow (\varphi \Rightarrow (\neg\varphi \Rightarrow \psi))$

The above axiom means that a contradiction (involving a formula φ) *plus* the information that φ is consistent (represented by the formula $\circ\varphi$, reading as "φ is consistent", where $\circ$ is the *consistency* connective) produce a logically trivial set. In other words, in order to have a trivial (or explosive) set of formulas Γ, it is *not* enough to derive a contradiction from Γ, that is:

$$\Gamma \vdash \varphi \quad \text{and} \quad \Gamma \vdash \neg\varphi \quad \text{does not imply that} \quad \Gamma \vdash \psi \ \text{ for every } \psi.$$

Indeed, in virtue of the modified version of the *Ex Contradictione Sequitur Quodlibet* property adopted by LFIs, the set Γ must also derive the consistency of the contradictory formula φ in order to be logically trivial, that is:

$$\Gamma \vdash \varphi \quad \text{and} \quad \Gamma \vdash \neg\varphi \ \text{ and } \ \Gamma \vdash \circ\varphi \quad \text{implies that} \quad \Gamma \vdash \psi \ \text{ for every } \psi.$$

The simplest LFI is the logic **mbC**, introduced in CARNIELLI, MARCOS, 2002 (see also CARNIELLI, CONIGLIO, MARCOS, 2007), which is presented below.

Definition 2.1 The calculus **mbC** is defined over the connectives $\{\neg, \wedge, \vee, \Rightarrow, \circ\}$ as follows:

Axiom schemas:

(Ax1) $\varphi \Rightarrow (\psi \Rightarrow \varphi)$

(Ax2) $(\varphi \Rightarrow \psi) \Rightarrow ((\varphi \Rightarrow (\psi \Rightarrow \gamma)) \Rightarrow (\varphi \Rightarrow \gamma))$

(Ax3) $\varphi \Rightarrow (\psi \Rightarrow (\varphi \wedge \psi))$

(Ax4) $(\varphi \wedge \psi) \Rightarrow \varphi$

(Ax5) $(\varphi \wedge \psi) \Rightarrow \psi$
(Ax6) $\varphi \Rightarrow (\varphi \vee \psi)$
(Ax7) $\psi \Rightarrow (\varphi \vee \psi)$
(Ax8) $(\varphi \Rightarrow \gamma) \Rightarrow ((\psi \Rightarrow \gamma) \Rightarrow ((\varphi \vee \psi) \Rightarrow \gamma))$
(Ax9) $\varphi \vee (\varphi \Rightarrow \psi)$
(Ax10) $\varphi \vee \neg\varphi$
(bc) $\circ\varphi \Rightarrow (\varphi \Rightarrow (\neg\varphi \Rightarrow \psi))$

Inference rule:
(MP) $\varphi, (\varphi \Rightarrow \psi) \therefore \psi$

It is worth noting that, if in **mbC** we substitute axiom (bc) (weak 'explosion law') by the classical 'explosion law'

(exp) $\varphi \Rightarrow (\neg\varphi \Rightarrow \psi)$

and if we remove the consistency connective o from the language, then we obtain an axiomatization of classical logic.

3. Standard Deontic Logic

Deontic logic is a modal logic designed for dealing with notions such as "it is obligatory that..." or "it is permitted (or permissible) that...". Usually, it is accomplished by extending the formal language with new connectives such as O, P and F, where the sentences Oφ, Pφ and Fφ denote that "φ is obligatory", "φ is permitted" and "φ is forbidden", respectively.

There exist several formal systems to deal with such operators, but the basic one is the so-called the *Standard Deontic Logic* (**SDL**), which is based on the classical paper VON WRIGHT, 1951. In this logic, it is possible to define P and F in terms of O, as we shall see below.

Now we briefly recall a presentation of **SDL**. From now on, *For* will denote the set of sentences generated by a given set Ξ of propositional variables and the connectives $\{\neg, \wedge, \vee, \Rightarrow, O\}$.

Definition 3.1 The calculus **SDL** is defined over the connectives $\{\neg, \wedge, \vee, \Rightarrow, O\}$ as follows:

Axiom schemas:

(Ax1)–(Ax10) from **mbC**, plus
(exp) $\varphi \Rightarrow (\neg\varphi \Rightarrow \psi)$
(O-K) $O(\varphi \Rightarrow \psi) \Rightarrow (O\varphi \Rightarrow O\psi)$
(O-E) $O\mathbf{f}_\varphi \Rightarrow \mathbf{f}_\varphi$ where $\mathbf{f}_\varphi =_{df} (\varphi \wedge \neg\varphi)$, for $\varphi \in \mathit{For}$

Inference rules:
(MP) $\varphi,\ (\varphi \Rightarrow \psi)\ \therefore\ \psi$
(O-NEC) $\vdash \varphi\ \therefore\ \vdash O\varphi$

It is worth noting that the *Deduction Metatheorem* (DM) holds in **SDL**:

(DM) $\Gamma, \varphi \vdash \psi$ if and only if $\Gamma \vdash \varphi \Rightarrow \psi$

Of course, the rule (O-NEC) can only be applied to theorems of **SDL**. Being so, $\varphi \vdash O\varphi$ does not hold in general. On the other hand, it is possible to perform *proof-by-cases* (PBC) in **SDL**:

(PBC) $\Gamma, \varphi \vdash \psi$ and $\Gamma, \neg\varphi \vdash \psi$ implies $\Gamma \vdash \psi$

We adopt from now on the usual abbreviation $(\varphi \Leftrightarrow \psi)$ to denote the formula

$$(\varphi \Rightarrow \psi) \wedge (\psi \Rightarrow \varphi).$$

It is well-known that, for every $\varphi, \psi \in \mathit{For}$, the following principles hold in **SDL**:

(O∧) $\vdash O(\varphi \wedge \psi) \Leftrightarrow (O\varphi \wedge O\psi)$

and

(O-exp) $O\varphi, O\neg\varphi \vdash \psi$.

On the other hand, if $P\varphi =_{df} \neg O \neg\varphi$ denotes the *permission* operator, where $P\varphi$ means that "φ is permissible", then the following principle holds in **SDL**:

(O-D) $O\varphi \vdash P\varphi$.

There exist several paradoxes in the literature concerning **SDL**. An important one is described below, which will be also analyzed in the view of the new proposed systems:

Example 3.2 *(Contrary-to-duty obligations)*

A well-known paradox of deontic logic is due to R. Chisholm (see CHISHOLM, 1963). The following formulation was presented in ÅQVIST, 2002 (see also the analysis in DUC, 1997). Consider the following sentences:

(1) It ought to be that John does not impregnate Suzy Mae.
(2) Not-impregnating Suzy Mae commits John to not marrying her.
(3) Impregnating Suzy Mae commits John to marry her.
(4) John impregnates Suzy Mae.

Let **i** and **m** be propositional constants representing the sentences "John impregnates Suzy Mae" and "John marries Suzy Mae", respectively. Clearly, (1) and (4) can be formalized as

(i) $O\neg\mathbf{i}$
(iv) $\mathbf{i}$

On the other hand, (2) can be formalized either as $O(\neg\mathbf{i} \Rightarrow \neg\mathbf{m})$ or $(\neg\mathbf{i} \Rightarrow O\neg\mathbf{m})$. Since the latter can be derived from (iv) (and since the four sentences above are supposed to be logically independent) we adopt the former formalization for (2):

(ii) $O(\neg\mathbf{i} \Rightarrow \neg\mathbf{m})$

With respect to (3), again two alternative formalizations are possible: either $O(\mathbf{i} \Rightarrow \mathbf{m})$ or $(\mathbf{i} \Rightarrow O\mathbf{m})$. Since the former is derived from (i), we adopt the latter:

(iii) $(\mathbf{i} \Rightarrow O\mathbf{m})$

Despite the set of four sentences (1)-(4) being apparently consistent, its formalization (i)-(iv) is inconsistent in **SDL**, because $O\neg\mathbf{m}$ is derived from (i) and (ii) by (O-K) and (MP), whereas $O\mathbf{m}$ is derived from (iii) and (iv) by (MP). From this, the set of sentences (i)-(iv) is logically trivial, by (O-exp).

It should be noted that the trivialization of the deontic operator O can already be obtained in a system weaker than **SDL**. In order to see this we begin by observing that, by removing the axiom schema (O-E) from **SDL,** it is obtained the modal logic **K**. Then, by (exp), (O-K) and transitivity of $\Rightarrow$ it follows that

$$\vdash O\varphi \Rightarrow (O\neg\varphi \Rightarrow O\psi)$$

holds in **K** for every φ and ψ. Therefore $\Gamma \mid\!- O\psi$ holds in **K** for every ψ, where Γ is the set of sentences (i)-(iv) of Example 3.2. In other words, the set Γ produces the trivialization of the notion of obligation O, even in the weaker system **K**.

There exist several alternatives for the purpose of avoiding the logic trivialization from the set Γ. A simple solution is to remove or modify either (exp) or (O-K), in order to block the derivation above. If one wants to keep classical logic as the basic framework, then (exp) cannot be removed, and thus a weaker version of axiom (O-K) could be considered. A different solution, which we develop in the following sections, is to keep (O-K) while considering a weaker version of (exp). More specifically, we propose the definition of a deontic logic based on LFIs, which are weaker than classical logic. By using a paraconsistent negation instead of a classical one, (exp) is no longer valid and so the trivialization argument above cannot be reproduced. It is worth noting that (O-E) does not play a crucial role in the trivialization argument above: it just guarantees the validity of (O-D), which is a desirable property of deontic systems, and then it should be maintained. As we shall see, (O-D) admits several formulations within a paraconsistent framework.

4. Logics of Deontic Inconsistency

Based on the idea of LFIs, we propose from now on deontic calculi weaker than **SDL**, based on LFIs instead of classical logic. That is, the negation $\neg$ can be assumed to be paraconsistent (and so weaker than classical negation), therefore contradictory obligations such as $O\varphi$ and $O\neg\varphi$ do not necessarily trivialize the system. Moreover, contradictory obligations do not trivialize the deontic operator O.

It is worth noting that the first approach to paraconsistent deontic logics in the literature was given in DA COSTA, CARNIELLI, 1986, where a deontic dimension was added to da Costa's well-known paraconsistent logic C_1 defining a system called C_1^D. This idea was additionally developed in PUGA, DA COSTA, CARNIELLI, 1988. Our proposal can be considered, in a certain sense, a generalization of DA COSTA, CARNIELLI, 1986. It should be mentioned that in CRUZ, 2005 another deontic extension of C_1 was proposed, but taking into consideration a stronger version of axiom (O-E), namely $O\varphi \Rightarrow \varphi$. Moreover, the operator O cannot be iterated, and so sentences such as $O(\varphi \Rightarrow O\psi)$ are not allowed. Due to these restrictions, some deontic paradoxes such as Chisholm's paradox cannot be satisfactorily treated in this approach. In COSTA-LEITE, 2003 it was also introduced a modal LFI called $\mathbf{Ci}^T$ in order to analyze Fitch's paradox of knowability. Recently, BUENO-SOLER, 2009 analyzed systems of positive modal logic as well as their extensions by adding paraconsistent negations satisfying properties of some LFIs, obtaining very general results of completeness and incompleteness. Additionally, new results derived from our present approach were obtained in PERON, 2009 and CONIGLIO, PERON, 2009.

The simplest deontic LFI is obtained from **mbC** (recall Definition 2.1) by adding a deontic operator O. Let For^{o} be the set of sentences generated by the set Ξ of propositional variables and the connectives $\neg$, $\wedge$, $\vee$, $\Rightarrow$, O, o.

Definition 4.1 The logic **DmbC** is defined over the connectives $\{\neg, \wedge, \vee, \Rightarrow, O, o\}$ by adding to the logic **mbC** (recall Definition 2.1) the following:

(O-K) $O(\varphi \Rightarrow \psi) \Rightarrow (O\varphi \Rightarrow O\psi)$
(O-E)^{o} $O\bot_{\varphi} \Rightarrow \bot_{\varphi}$ where $\bot_{\varphi} =_{df} (\varphi \wedge \neg\varphi) \wedge o\varphi$, for $\varphi \in For^{o}$
(O-NEC) $\vdash \varphi \;\therefore\; \vdash O\varphi$

It should be noted that $\mathbf{f}_{\varphi} = (\varphi \wedge \neg\varphi)$ does not trivialize **DmbC**, as in the case of classical logic (as well as in **SDL**): there exist φ and ψ such that ψ does not follow from $\mathbf{f}_{\varphi}$ in **DmbC**. On the other hand, $\bot_{\varphi} \vdash \psi$ for every φ and ψ in For^{o}, by (bc). Clearly **DmbC** enjoys (DM) and (PBC).

The following notion of *deontic inconsistency* is useful for our purposes:

Definition 4.2 The expression $\otimes\varphi$ will stand for the formula $O\neg o\varphi$, meaning that φ is 'deontically unsafe', or φ is 'deontically ill-behaved', or even that φ is 'deontically inconsistent'.

It is immediate to see that contradictory obligations allow to infer that the involved sentence is deontically inconsistent:

Proposition 4.3 In **DmbC** the following holds:

$$\vdash O\varphi \Rightarrow (O\neg\varphi \Rightarrow \otimes\varphi).$$

Proof: It is easy to prove that

$$\vdash \varphi \Rightarrow (\neg\varphi \Rightarrow \neg o\varphi)$$

because it is valid in **mbC** (see, for instance, CARNIELLI, CONIGLIO, MARCOS, 2007). Then, using (O-NEC),

$$\vdash O(\varphi \Rightarrow (\neg\varphi \Rightarrow \neg o\varphi))$$

and so, by (O-K), (MP), transitivity of $\Rightarrow$ and the definition of $\otimes$, we get the desired result.

Q.E.D.

Moreover, in **DmbC** contradictory obligations do not trivialize the operator O and, therefore, do not trivialize the logic: it is not always the case that

$$O\varphi,\ O\neg\varphi \vdash O\psi$$

(by taking, for instance, $\varphi = p$ and $\psi = q$, where p and q are distinct propositional variables). Therefore, it is not always the case that

$$O\varphi,\ O\neg\varphi \vdash \delta$$

This can be easily proved by using Kripke structures, to be defined in Section 5 below (see Proposition 5.3).

The next example illustrates how the proposed notion of deontic inconsistency appears in deontic paradoxes.

Example 4.4 *(Contrary-to-duty obligations, cont.)*
Recall Example 3.2 concerning Chisholm's paradox. Let **i** and **m** be propositional constants representing the sentences "John impregnates Suzy Mae" and "John marries Suzy Mae", and let Γ be the following set of sentences:

(i) $O\neg\mathbf{i}$
(ii) $O(\neg\mathbf{i} \Rightarrow \neg\mathbf{m})$
(iii) $(\mathbf{i} \Rightarrow O\mathbf{m})$
(iv) $\mathbf{i}$

Then, the sentence $O\neg\mathbf{m}$ is derived in **DmbC** from (i) and (ii), whereas $O\mathbf{m}$ is derived in **DmbC** from (iii) and (iv). Thus, by Proposition 4.3, the sentence $\otimes\mathbf{m}$ is derived from Γ in **DmbC**, without trivializing the system. That is, the logic **DmbC** allows to infer from (i)-(iv) that the sentence "John marries Suzy Mae" is deontically inconsistent, instead of trivializing (as in the case of **SDL** or **K**).

As mentioned in the Introduction, the derivation in Example 4.4 of deontic inconsistencies concerning the sentences in Γ do not solve Chisholm's paradox. However, the fact that sentences of the form $\otimes\varphi$ can be derived from a logically trivial set (w.r.t. **SDL**) helps to detect a deontic conflict. From this information, such sentences could be, for instance, removed or suitably modified in order to overcome the conflict.

5. Kripke semantics for DmbC

In this section it is presented a Kripke-style adequate semantics for **DmbC**. It is an adaptation of the Kripke semantics for modal paraconsistent logic $\mathbf{Ci}^{T}$ introduced in COSTA-LEITE, 2003. As mentioned above, a very general completeness result for modal LFIs and positive modal logics was recently obtained in BUENO-SOLER, 2009.

From now on, **2** will denote the set $\{0,1\}$ of classical truth-values.

Definition 5.1 A *Kripke structure* for **DmbC** is a triple $\langle W,R,\{V_w\}_{w\in W}\rangle$, where:

1. W is a non-empty set (of *possible-worlds*);
2. $R \subseteq W\times W$ is a relation (of *accessibility*) between possible-worlds which is *serial*, that is: for every $w\in W$ there exists $w'\in W$ such that wRw';
3. V_w is a mapping from For^{o} to *2* satisfying the following clauses, for each $w\in W$:
   - (v1) $V_w(\varphi \wedge \psi) = 1$ iff $V_w(\varphi) = V_w(\psi) = 1$;
   - (v2) $V_w(\varphi \vee \psi) = 0$ iff $V_w(\varphi) = V_w(\psi) = 0$;
   - (v3) $V_w(\varphi \Rightarrow \psi) = 0$ iff $V_w(\varphi) = 1$ and $V_w(\psi) = 0$;
   - (v4) $V_w(\varphi) = 0$ implies that $V_w(\neg\varphi) = 1$;
   - (v5) $V_w(\varphi) = V_w(\neg\varphi)$ implies that $V_w(\mathrm{o}\varphi) = 1$;
   - (v6) $V_w(\mathrm{O}\varphi) = 1$ iff $V_{w'}(\varphi) = 1$ for every w' such that wRw'.

Given a Kripke structure $M = \langle W,R,\{V_w\}_{w\in W}\rangle$ for **DmbC**, a world w in W and a formula φ in For^{o}, we write $M,w \models \varphi$ to denote that $V_w(\varphi) = 1$. If $\Gamma \subseteq For^{o}$ then the notation $M,w \models \Gamma$ will stand for $M,w \models \gamma$, for every $\gamma \in \Gamma$. The notion of semantical consequence within Kripke structures for **DmbC** is defined as expected. Thus, given a finite set $\Gamma \cup \{\varphi\} \subseteq For^{o}$, we say that φ *follows semantically from* Γ *in* **DmbC**, written $\Gamma \models \varphi$, if, for every Kripke structure M for **DmbC** and every $w\in W$: $M,w \models \Gamma$ implies $M,w \models \varphi$.

Soundness of **DmbC** with respect to Kripke structures for **DmbC** follows straightforwardly.

Theorem 5.2 *(Soundness for* **DmbC***)*
Let $\Gamma \cup \{\varphi\}$ be a finite set of formulas in For^{o}. Then $\Gamma \vdash \varphi$ implies that $\Gamma \models \varphi$.

Proof: It is enough to prove that $M,w \models \varphi$ for every axiom φ of **DmbC** and every Kripke structure M and world w; together with this, it must be proved the soundness of the inference rules. The unique axioms deserving attention are (bc) and (O-E)º since the others are clearly valid. The fact that $M,w \models \varphi$ for every instance φ of (bc) is an easy consequence of clauses (v3) and (v5) of Definition 5.1. With respect to (O-E)º, suppose that $V_w(\mathrm{O}\bot_\varphi) = 1$. Let $w'\in W$ such that wRw' (such a w' exists, because R is serial). Thus $V_{w'}(\bot_\varphi) = 1$ and then $V_{w'}(\varphi) = V_{w'}(\neg\varphi) = V_{w'}(\mathrm{o}\varphi) = 1$, which contradicts clause (v5) of Definition 5.1. Therefore $V_w(\mathrm{O}\bot_\varphi) = 0$ and so $V_w(\mathrm{O}\bot_\varphi \Rightarrow \bot_\varphi) = 1$, by (v3). With respect to (MP), it is clear that, if $M,w \models \varphi \Rightarrow \psi$ and $M,w \models \varphi$ then $M,w \models \psi$, by clause (v3).

With respect to (O-NEC), if $M,w \models \varphi$ for every M,w then, by clause (v6), $M,w \models \mathrm{O}\varphi$ for every M and w.

Q.E.D.

Using soundness of **DmbC** with respect to Kripke structures, it is easy to show that contradictory obligations do not trivialize:

Proposition 5.3 Let p and q be different propositional variables. Then, in **DmbC**:
(i) It is not the case that $\mathrm{O}p, \mathrm{O}\neg p \vdash \mathrm{O}q$;
(ii) It is not the case that $\mathrm{O}p, \mathrm{O}\neg p \vdash q$.

Proof: Let p and q be different propositional variables. Consider the Kripke structure $M = \langle W,R,\{V_w\}_{w\in W}\rangle$ for **DmbC** such that $W = \{w\}$, $R = \{\langle w,w\rangle\}$ and V_w is the extension to For° of the mapping $V\colon \Xi \cup \{\neg p \ : \ p \in \Xi\} \to \mathbf{2}$ such that $V(p) = V(\neg p) = 1$, $V(q) = 0$, and $V(\neg r) = 1$ iff $V(r) = 0$ for every $r \in \Xi$ different from p. The mapping V_w can be easily obtained inductively from V by defining:

(a) $V_w(\neg\varphi) = 1$ iff $V_w(\varphi) = 0$;
(b) $V_w(\mathrm{O}\varphi) = V_w(\varphi)$;
(c) $V_w(\circ\varphi) = 1$ iff $V_w(\varphi) \neq V_w(\neg\varphi)$ (provided that the complexity of $\circ\varphi$ is defined to be greater than the complexity of $\neg\varphi$); and
(d) $V_w(\varphi\#\psi)$ is defined according to clauses (v1)-(v3) of Definition 5.1, for every connective $\# \in \{\wedge, \vee, \Rightarrow\}$.

Then $M,w \models \{\mathrm{O}p, \mathrm{O}\neg p\}$ but it is not the case that $M,w \models \mathrm{O}q$ and so $\mathrm{O}q$ is not a semantical consequence of $\{\mathrm{O}p, \mathrm{O}\neg p\}$ in **DmbC**. Therefore $\mathrm{O}q$ is not a derivable from $\{\mathrm{O}p, \mathrm{O}\neg p\}$ in **DmbC**, by Theorem 5.2. Analogously (by using the same Kripke structure) it is proved that q is not a derivable from $\{\mathrm{O}p, \mathrm{O}\neg p\}$ in **DmbC**.

Q.E.D.

On the other hand, with additional hypothesis concerning consistency, contradictory obligations trivialize **DmbC**. In order to see this, we introduce the following notion of *deontic consistency*:

Definition 5.4 The expression $\oplus\varphi$ will stand for the formula $\mathrm{O}\circ\varphi$, meaning that φ is ‘deontically safe’, or that φ is ‘deontically well-behaved’, or even that φ is ‘deontically consistent’.

Proposition 5.5 In **DmbC** it holds

$$\mathrm{O}\varphi, \mathrm{O}\neg\varphi, \oplus\varphi \vdash \bot_\varphi$$

and then

$$\mathrm{O}\varphi, \mathrm{O}\neg\varphi, \oplus\varphi \vdash \psi$$

for every φ, ψ.

Proof: From (bc) (recall Definition 2.1), (O-K), (MP) and transitivity of $\Rightarrow$, it follows that $\vdash \mathrm{O}\circ\varphi \Rightarrow (\mathrm{O}\varphi \Rightarrow (\mathrm{O}\neg\varphi \Rightarrow \mathrm{O}\bot_{\varphi})$ and so, by (MP),

$$\mathrm{O}\varphi,\ \mathrm{O}\neg\varphi,\ \oplus\varphi \vdash \mathrm{O}\bot_{\varphi}.$$

Then, by (O-E)$^{\circ}$, (MP) and definition of $\oplus$ it follows that

$$\mathrm{O}\varphi,\ \mathrm{O}\neg\varphi,\ \mathrm{O}\circ\varphi \vdash \bot_{\varphi}.$$

Finally,

$$\mathrm{O}\varphi,\ \mathrm{O}\neg\varphi,\ \mathrm{O}\circ\varphi \vdash \psi$$

since $\bot_{\varphi} \vdash \psi$ for every ψ.

Q.E.D.

In order to prove the completeness theorem for **DmbC** with respect to its Kripke structures, a canonical model can be constructed. Firstly, the notion of φ-saturated set in **DmbC** is considered.

Definition 5.6 Let $\Delta \cup \{\varphi\} \subseteq For^{\circ}$ be a set of formulas. We say that Δ *is* φ-*saturated in* **DmbC** if:

(i) it is not the case that $\Delta \vdash \varphi$ in **DmbC**;
(ii) if $\psi \notin \Delta$ then $\Delta, \psi \vdash \varphi$ in **DmbC**.

The following properties can be easily proved:

Lemma 5.7 Let Δ be a φ-saturated set in **DmbC**. Then:

(i) $\Delta \vdash \psi$ iff $\psi \in \Delta$;
(ii) $(\delta \wedge \gamma) \in \Delta$ iff $\delta \in \Delta$ and $\gamma \in \Delta$;
(iii) $(\delta \vee \gamma) \in \Delta$ iff either $\delta \in \Delta$ or $\gamma \in \Delta$;
(iv) $(\delta \Rightarrow \gamma) \in \Delta$ iff either $\delta \notin \Delta$ or $\gamma \in \Delta$;
(v) if $\psi \notin \Delta$ then $\psi \in \Delta$;
(vi) if $\psi, \neg\psi \in \Delta$ then $\circ\psi \in \Delta$.

The following version of Lindenbaum-Asser's Lemma can be proved, by adapting the classical proof (see a general result in CARNIELLI, CONIGLIO, MARCOS, 2007):

Lemma 5.8 Let $\Delta \cup \{\varphi\} \subseteq For^{\circ}$ be a set of formulas such that, in **DmbC,** it is not the case that $\Delta \vdash \varphi$. Then, there exists a φ-saturated set Δ' in **DmbC** such that $\Delta \subseteq \Delta'$.

The following notion will be useful:

Definition 5.9 Let Δ be a φ-saturated set in **DmbC**. The *denecessitation* of Δ is the set $Den(\Delta) =_{df} \{\psi \in For^{\circ} : O\psi \in \Delta\}$.

In order to obtain the completeness theorem, one additional technical lemma is needed.

Lemma 5.10 Let Δ be a φ-saturated set in **DmbC**.

(i) The set $Den(\Delta)$ is a closed theory in **DmbC**, that is: if $Den(\Delta) \vdash \psi$ in **DmbC** then $\psi \in Den(\Delta)$.

(ii) If $O\psi \notin \Delta$ then it is not the case that $Den(\Delta), \neg\psi \vdash \psi$.

Proof: (i) Suppose that $Den(\Delta) \vdash \psi$ in **DmbC**. Then, there exist $\psi_1,\ldots, \psi_n \in Den(\Delta)$ such that $\psi_1,\ldots, \psi_n \vdash \psi$ in **DmbC** and so, by (DM), it follows that

$$\vdash (\psi_1 \Rightarrow (\ldots \Rightarrow (\psi_n \Rightarrow \psi) \ldots)).$$

By (O-NEC), $\vdash O(\psi_1 \Rightarrow (\ldots \Rightarrow (\psi_n \Rightarrow \psi) \ldots))$ and then, by (O-K), (MP) and transitivity of $\Rightarrow$, we get $\vdash (O\psi_1 \Rightarrow (\ldots \Rightarrow (O\psi_n \Rightarrow O\psi) \ldots))$. But $O\psi_1,\ldots, O\psi_n \in \Delta$, by definition of $Den(\Delta)$, therefore $\Delta \vdash O\psi$, by (MP). Thus $O\psi \in \Delta$, by property (i) of Lemma 5.7, and then $\psi \in Den(\Delta)$.

(ii) Suppose that $Den(\Delta),\neg\psi \vdash \psi$ holds in **DmbC**. Since $Den(\Delta), \psi \vdash \psi$ then, using (PBC), it follows that $Den(\Delta) \vdash \psi$. Thus, by item (i), $\psi \in Den(\Delta)$. That is, $O\psi \in \Delta$.

Q.E.D.

Definition 5.11 The *canonical model* for **DmbC** is the triple

$$M_c = \langle W, R, \{V_\Delta\}_{\Delta \in W} \rangle$$

Such that:

(i) $W = \{\Delta \subseteq For^{\circ} : \Delta$ is a φ-saturated set in **DmbC** for some $\varphi\}$;

(ii) $R = \{\langle \Delta,\Delta' \rangle \in W \times W : Den(\Delta) \subseteq \Delta' \}$;

(iii) V_Δ is the characteristic map of Δ, that is: $V_\Delta(\psi) = 1$ iff $\psi \in \Delta$.

Using the lemmas stated above it is easy to prove the following:

Proposition 5.12 The canonical model M_c is a Kripke structure for **DmbC**.

Proof: We begin by proving that R is serial. Thus, let Δ be a φ-saturated set in **DmbC**. Then, there must be a formula ψ such that it is not the case that $Den(\Delta) \vdash \psi$. Otherwise, $Den(\Delta) \vdash \bot_{\varphi}$ and then $\bot_{\varphi} \in Den(\Delta)$, by Lemma 5.10 (i). Thus $O\bot_{\varphi} \in \Delta$ and so $\Delta \vdash \bot_{\varphi}$, by (O-E)° and (MP). From this it follows that $\Delta \vdash \varphi$, a contradiction. Therefore, there must be some formula ψ such that it is not the case that $Den(\Delta) \vdash \psi$ in **DmbC**. Then, by Lemma 5.8, there exists a ψ-saturated set Δ' in **DmbC** such that $Den(\Delta) \subseteq \Delta'$. In other words, there exists $\Delta' \in W$ such that $\Delta R \Delta'$, and then R is serial.

Let $\Delta \in W$. By Lemma 5.7 (ii)-(vi) it follows that V_{Δ} satisfies clauses (v1)-(v5) of Definition 5.1. It remains to prove that, for every formula ψ in For°:

$$V_{\Delta}(O\psi) = 1 \text{ iff } V_{\Delta'}(\psi) = 1 \text{ for every } \Delta' \text{ such that } Den(\Delta) \subseteq \Delta'.$$

Let ψ such that $V_{\Delta}(O\psi) = 1$ and let $\Delta' \in W$ such that $Den(\Delta) \subseteq \Delta'$. Then $O\psi \in \Delta$, and so $\psi \in Den(\Delta)$, by definition of $Den(\Delta)$. Therefore $\psi \in \Delta'$ and then $V_{\Delta'}(\psi) = 1$.

Conversely, if $V_{\Delta}(O\psi) = 0$ then $O\psi \notin \Delta$. Thus, by Lemma 5.10(ii), it is not the case that $Den(\Delta), \neg\psi \vdash \psi$. Using Lemma 5.8, there exists a ψ-saturated set Δ' in **DmbC** such that $Den(\Delta) \cup \{\neg\psi\} \subseteq \Delta'$. Therefore $\Delta' \in W$ such that $Den(\Delta) \subseteq \Delta'$ and $V_{\Delta'}(\psi) = 0$. Thus, $\{V_{\Delta}\}_{\Delta \in W}$ satisfies clause (v6) of Definition 5.1 and then M_c is in fact a Kripke structure for **DmbC**.

Q.E.D.

We can finally prove the completeness theorem for **DmbC**:

Theorem 5.13 *(Completeness for* **DmbC***)* Let $\Gamma \cup \{\varphi\}$ be a set of formulas in For°. Then **DmbC** satisfies: $\Gamma \models \varphi$ implies that $\Gamma \vdash \varphi$.

Proof: Suppose that it is not the case that $\Gamma \vdash \varphi$. By Lemma 5.8, we can extend Γ to a φ-saturated set Δ in **DmbC**. Since it is not the case that $\Delta \vdash \varphi$ then $\varphi \notin \Delta$. Let M_c be the canonical model for **DmbC** (cf. Definition 5.11). Then, by Proposition 5.12, M_c is a Kripke structure for **DmbC** and Δ is a possible-world of M_c such that $M_c, \Delta \models \Gamma$ (since $\Gamma \subseteq \Delta$) and it is not the case that $M_c, \Delta \models \varphi$ (since $\varphi \notin \Delta$). This shows that it is not the case that $\Gamma \models \varphi$ in **DmbC**.

Q.E.D.

6. Permission and Prohibition

As mentioned above, in **SDL** the permission operator P and the prohibition operator F can be defined in terms of the obligation operator O. In fact, the following interactions can be stated:

$$P\varphi \equiv \neg O\neg\varphi \quad \text{and} \quad F\varphi \equiv O\neg\varphi$$

and so $P\varphi \equiv \neg F\varphi$. Of course, the definition above of P and F in terms of O is strongly based on the fact that $\neg$ is a classical negation in **SDL**. When $\neg$ is substituted by a weaker negation such as the paraconsistent one of **DmbC** the situation changes, and so weaker deontic notions are obtained from those definitions. However, the use of an appropriate strong negation (definable in **DmbC**) can restore the desired properties of the deontic operators defined by the equivalences above.

The rest of this section is devoted to analyze the definition of the deontic *permissible* and *forbidden* operators in the logic **DmbC**. Additionally, several forms of the law

$$(*) \quad O\varphi \Rightarrow P\varphi$$

of **SDL** will be analyzed in the context of logic **DmbC**. Observe that, in **SDL**, (*) is equivalent to (O-D).

In order to analize (*) in the paraconsistent framework, recall the notion of *deontic consistency* (cf. Definition 5.4).

We begin by observing that, in **mbC** (and so, in **DmbC**) it is possible to define a classical negation ~ as follows: $\sim\varphi =_{df} \varphi \Rightarrow \bot_\varphi$. The derived connective ~ plays the role of a classical negation because of the following:

Proposition 6.1 For every $\varphi, \psi \in For^{\circ}$ the following properties hold in **DmbC**:

(i) $\varphi, \sim\varphi \vdash \psi$;
(ii) $\vdash (\varphi \vee \sim\varphi)$;
(iii) $\sim\sim\varphi \vdash \varphi$;
(iv) $\varphi \vdash \sim\sim\varphi$.

Proof: It follows from the corresponding proof for the negation ~ defined analogously in **mbC** (cf. CARNIELLI, CONIGLIO, MARCOS, 2007).

Q.E.D.

Therefore, the logic **DmbC** has two negations: a paraconsistent one, $\neg$, and a classical one, ~. Being so, it is possible to define in **DmbC** four *permissibility operators* from O by combining both negations as follows:

(P1) $P_1\varphi =_{df} \neg O\neg\varphi$;
(P2) $P_2\varphi =_{df} \sim O\neg\varphi$;
(P3) $P_3\varphi =_{df} \neg O\sim\varphi$;
(P4) $P_4\varphi =_{df} \sim O\sim\varphi$.

The formula $P_i\varphi$ reads as "φ is i-permissible", for i = 1,…,4.

Analogously, it is possible to define in **DmbC** two *prohibition operators* from O by using both negations as follows:

(F1) $F_1\varphi =_{df} O\neg\varphi$;

(F2) $F_2\varphi =_{df} O{\sim}\varphi$.

The formula $F_i\varphi$ reads as "φ is i-forbidden", for i = 1, 2.

The semantics of these operators works "classically" just in the case of P_4 and F_2; in the case of P_2 just a half of the classical definition of permissibility holds, and in the case of P_1, P_3 and F_1 the semantics is far from classical, because of the paraconsistent characteristics of ¬. For instance, it can be easily verified that

- $M, w \models P_4\varphi$ if and only if $M, w' \models \varphi$ for some $w' \in W$ such that wRw';
- $M, w \models F_2\varphi$ if and only if, for every $w' \in W$ such that wRw', it is not the case that $M, w' \models \varphi$;
- $M, w \models P_2\varphi$ implies that $M, w' \models \varphi$ for some $w' \in W$ such that wRw'.

Various of the i-permission operators P_i and the i-forbidden operators F_i have not clear meaning. However, some interpretations are still possible. For instance, while P_4 and F_2 correspond to the classical notion of permission and prohibition, respectively, P_2 and F_1 could be interpreted as representing a kind of *prima facie* permission and a *prima facie* prohibition, respectively.

With respect to the law (∗) stating the derivation of permissibility from obligation, it can be proven the following versions of (O-D) in **DmbC**:

Proposition 6.2 In **DmbC** the following holds, for every $\varphi \in For^\circ$:

(i) $\oplus\varphi \vdash O\varphi \Rightarrow P_1\varphi$

(ii) $\oplus\varphi \vdash O\varphi \Rightarrow P_2\varphi$

(iii) $\vdash O\varphi \Rightarrow P_3\varphi$

(iv) $\vdash O\varphi \Rightarrow P_4\varphi$

Proof: (i) From Proposition 5.5 it follows that in **DmbC** it holds

$$O\varphi, \oplus\varphi, O\neg\varphi \vdash P_1\varphi.$$

On the other hand, by definition of P_1,

$$O\varphi, \oplus\varphi, \neg O\neg\varphi \vdash P_1\varphi$$

and so, using (PBC),

$$O\varphi, \oplus\varphi \vdash P_1\varphi.$$

The rest of the proof follows by (DM).
(ii) From Proposition 5.5 it follows that

$$O\varphi, \oplus\varphi, O\neg\varphi \vdash \bot_\psi$$

for $\psi = O\neg\varphi$. The rest of the proof follows by (DM) and the definition of P_2.
(iv) Since $\varphi, {\sim}\varphi \vdash \bot_\varphi$ it follows (as usual) that

$$O\varphi, O{\sim}\varphi \vdash O\bot_\varphi.$$

By (O-E)° and the fact that $\bot_\varphi \vdash P_3\varphi$ we get

$$O\varphi, O{\sim}\varphi \vdash P_3\varphi.$$

On the other hand, by definition of P_3,

$$O\varphi, \neg O{\sim}\varphi \vdash P_3\varphi$$

and so $O\varphi \vdash P_3\varphi$, by (PBC). The rest of the proof follows by (DM).
(iv) As proved above, $O\varphi, O{\sim}\varphi \vdash O\bot_\varphi$. Using that $\bot_\varphi \vdash \bot_\psi$ for $\psi = O{\sim}\varphi$ it follows that $O\varphi \vdash {\sim}O{\sim}\varphi$, by (DM) and definition of ~. The rest of the proof follows by (DM) and the definition of P_4.

Q.E.D.

With respect to the *prohibition* operators, the following proposition is easy to prove. We left to the reader the details of the proof.

Proposition 6.3 Let $\varphi, \psi \in For^\circ$ and let p, q be two different propositional variables. Then the following hold in **DmbC**:

(i) $\oplus\varphi, O\varphi, F_1\varphi \vdash \varphi$
(ii) It is not the case that $Op, F_1p \vdash q$
(iii) $O\varphi, F_2\varphi \vdash \psi$.

7. Propagating deontic inconsistency and deontic consistency

Finally, we present in this section another deontic LFI, which is stronger than **mbC**. It is based on the logic **LFI1** introduced in CARNIELLI, MARCOS, DE AMO, 2000, which allows inconsistency to be propagated. In that paper, a first-order version of **LFI1** was used for dealing with databases allowing contradictions.

From now on, the expression $\bullet\varphi$ will stand for the formula $\neg\circ\varphi$, denoting the *inconsistency* (or *non-consistency*) of φ. Recalling the deontic inconsistency operator $\otimes$ introduced in Definition 3.2, it can now be written as $\otimes\varphi = O\bullet\varphi$.

Definition 7.1 The logic **LFI1** is obtained by adding to **mbC** the following axiom schemas:

(cef) $\varphi \Leftrightarrow \neg\neg\varphi$
(ci) $\bullet\varphi \Rightarrow (\varphi \wedge \neg\varphi)$
(cj1) $\bullet(\varphi \wedge \psi) \Leftrightarrow ((\bullet\varphi \wedge \psi) \vee (\bullet\psi \wedge \varphi))$
(cj2) $\bullet(\varphi \vee \psi) \Leftrightarrow ((\bullet\varphi \wedge \neg\psi) \vee (\bullet\psi \wedge \neg\varphi))$
(cj3) $\bullet(\varphi \Rightarrow \psi) \Leftrightarrow (\varphi \wedge \bullet\psi)$

Definition 7.2 The logic **DLFI1** is defined by adding to the logic **LFI1** the following:

(O-K) $O(\varphi \Rightarrow \psi) \Rightarrow (O\varphi \Rightarrow O\psi)$
(O-E)° $O\bot_\varphi \Rightarrow \bot_\varphi$ where $\bot_\varphi =_{df} (\varphi \wedge \neg\varphi) \wedge \circ\varphi$, for $\varphi \in For^\circ$
(O-NEC) $\vdash \varphi \;\; \therefore \;\; \vdash O\varphi$

Equivalently, **DLFI1** can be defined as the logic obtained from **DmbC** by adding the new axiom schemas of Definition 7.1.

The proof of the following properties of "propagation of deontic inconsistency" in **DLFI1** is straightforward:

Proposition 7.3 The logic **DLFI1** satisfies the following:

(i) $\vdash \otimes\varphi \Leftrightarrow (O\varphi \wedge O\neg\varphi)$
(ii) $\otimes\varphi, O\psi \vdash \otimes(\varphi \wedge \psi)$
(iii) $\otimes\psi, O\varphi \vdash \otimes(\varphi \wedge \psi)$
(iv) $\otimes\varphi, O\neg\psi \vdash \otimes(\varphi \vee \psi)$
(v) $\otimes\psi, O\neg\varphi \vdash \otimes(\varphi \vee \psi)$
(vi) $\vdash (O\varphi \wedge \otimes\psi) \Leftrightarrow \otimes(\varphi \Rightarrow \psi)$

An adequate Kripke semantics for **DLFI1** can be defined by adding the obvious clauses to the mappings V_w representing the new axioms from Definition 7.1. We left the details to the reader.

It is interesting to note that, because of the propagation of deontic inconsistency, the Chisholm paradox produces new sentences which are deontically inconsistent in **DLFI1**:

Example 7.4 *(Contrary-to-duty obligations, cont.)*
Recall Example 3.2. Using Example 4.4, from the set Γ of sentences

(i) $O\neg\mathbf{i}$
(ii) $O(\neg\mathbf{i} \Rightarrow \neg\mathbf{m})$
(iii) $(\mathbf{i} \Rightarrow O\mathbf{m})$
(iv) $\mathbf{i}$

it is derived in the logic **DmbC** (and then, in **DLFI1**) the sentence $\otimes\mathbf{m}$. Since, in **DLFI1,** $\Gamma \vdash (O\neg\mathbf{i} \wedge \otimes\mathbf{m})$, it follows from Proposition 7.3 that

$$\Gamma \vdash \otimes(\neg\mathbf{i} \wedge \mathbf{m}),$$
$$\Gamma \vdash \otimes(\mathbf{i} \vee \mathbf{m}),$$
$$\Gamma \vdash \otimes(\neg\mathbf{i} \Rightarrow \mathbf{m}).$$

That is, new sentences are proved to be deontically inconsistent from Γ, by using a logic system stronger than **DmbC**.

8. Final Remarks

This paper proposes a logic context in which conflicting obligations are allowed without trivializing the system. The key is to use a logic basis weaker than classical logic and tolerant to contradictions, that is, a paraconsistent logic. Being so, the present framework is shown to be a suitable framework for analyzing moral dilemmas and deontic paradoxes.

As pointed out above, a similar line of research was introduced in DA COSTA, CARNIELLI, 1986 (see also PUGA, DA COSTA, CARNIELLI, 1988); however, there exist some differences between those proposals and our approach. In the former references the proposed system, called C_1^D, is based on paraconsistent logic C_1 in which the consistency operator can be defined in terms of the other connectives as follows: $\circ\varphi := \neg(\varphi \wedge \neg\varphi)$. Besides this, consistency "propagates" through the connectives (including O) and so, in particular, $\circ\varphi \Rightarrow \circ O\varphi$ is a theorem of C_1^D. Moreover, the formula $O\varphi \Rightarrow \neg O\neg\varphi$ is also a theorem of C_1^D which, as we saw in Proposition 6.2 (i), is not the case in **DmbC**: in order to obtain $P_1\varphi$ from $O\varphi$ in **DmbC** it is necessary to add the hypothesis that φ is deontically consistent, that is, $\oplus\varphi$. Besides this, the logic basis **mbC** is strictly weaker than C_1 and so we could say that the present approach is slightly more general than that of DA COSTA, CARNIELLI, 1986.

As shown in the last section, the stronger system **DLFI1** allows to deal with more sophisticated notions involving deontic consistency and deontic inconsistency, within a richer language in which deontic and consistency operators interact naturally. Additionally, this system could shed some light on the analysis of deontic inconsistencies in the context of databases. The extension of **DLFI1** to first-order logic by adapting the semantics presented in CARNIELLI, MARCOS, DE AMO, 2000 is the first step towards this goal.

In PERON, 2009 and in CONIGLIO, PERON, 2009 the concept of Logics of Deontic Inconsistency was generalized and some applications to the analysis of deontic paradoxes were obtained. The point of view for the analysis, however, was slightly different to the present one: the emphasis was given on the possibility of representing *more* sentences in richer a logical language which contains two negations (a classical and a paraconsistent one), and where several logical dependencies (typical of classical logic) disappear. This allows to formalize paradoxes such as Chisholm paradox in several different ways, dissolving so the paradox. This perspective of logics of deontic consistency, together with the approach we gave in this paper, shows the potentialities of this kind of logics for the study of deontic paradoxes.

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