Deontic Logic (Adapted) for Normative Conflicts

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Part I

— THE PROBLEM OF NORMATIVE CONFLICTS —

Deontic Logic — the logic of ‘ought’.

Assume a monadic propositional deontic logic — Read OA as ‘it ought to be that A’, or ‘the agent ought to do A —

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Normative Conflicts — the agent ought to do each of several things but cannot do them all.

Strict, or simple, conflicts — OA, $O\neg A$

Binary conflicts — OA, OB, $\vdash \neg(A \wedge B)$

n-ary conflicts (the generic form) — $OA_1, \ldots, OA_n$, $\vdash \neg(A_1 \wedge \cdots \wedge A_n)$

e.g., $O(A \vee B)$, $O\neg A$, $O\neg B$

— Standard Principles —

No-conflict	(D)	$OA \rightarrow \neg O \neg A$
Ought $\Rightarrow$ Can	(P)	$\neg O(A \wedge \neg A)$

Principles of focus:

base logic	CL	classical propositional logic, including
ex contradictione quodlibet	(ECQ)	$(A \wedge \neg A) \rightarrow B$
Aggregation	(C)	$(OA \wedge OB) \rightarrow O(A \wedge B)$
Distribution	(RM)	If $\vdash A \rightarrow B$ then $\vdash OA \rightarrow OB$, or
Simplification	(M)	$O(A \wedge B) \rightarrow OA$, and
Replacement	(RE)	If $\vdash A \leftrightarrow B$ then $\vdash OA \leftrightarrow OB$

Others to be mentioned:

K-principle	(K)	$O(A \rightarrow B) \rightarrow (OA \rightarrow OB)$
Belnap's principle	(Bel)	$O(A \vee B) \rightarrow (\neg O \neg A \vee OB)$
Necessitation	(N)	If $\vdash A$ then $\vdash OA$

SDL = **KD** = [CL + (C) + (RM) + (N) + (D)] =
[CL + (C) + (M) + (RE) + (N) + (P)] =
[CL + (K) + (N) + (D)]

K = [CL + (C) + (RM) + (N)] =
[CL + (C) + (M) + (RE) + (N)] =
[CL + (K) + (N)]

— Deontic Explosion —

The simplest form — (DEX) — $\vdash (OA \wedge O\neg A) \rightarrow OB$

Proof —

i)	$OA \wedge O\neg A$	hyp
ii)	$\vdash (A \wedge \neg A) \rightarrow B$	CL (ECQ)
iii)	$\vdash O(A \wedge \neg A) \rightarrow OB$	ii, (RM)
iv)	$O(A \wedge \neg A)$	i, (C)
v)	OB	iii, iv, *modus ponens*

— Classically-based non-explosive deontic logics —

a) Non-adjunctive systems, reject (C)

— MDL = **P** = CL + (RM) + (N)

b) Non-distributive systems, reject (RM)

— **ECN** = CL + (RE) + (C) + (N)

c) Non-aggregative, non-distributive systems, reject both (C) and (RM)

— **EN** = CL + (RE) + (N)

— Paraconsistent systems —

i) Basic paraconsistent propositional logic, **PL**, including

Id)	$A \to A$
$\wedge$E)	$(A \wedge B) \to A$
$\wedge$E)	$(A \wedge B) \to B$
$\wedge$I)	$((A \to B) \wedge (A \to C)) \to (A \to (B \wedge C))$
$\vee$I)	$A \to (A \vee B)$
$\vee$I)	$B \to (A \vee B)$
$\vee$E)	$((A \to C) \wedge (B \to C)) \to ((A \vee B) \to C)$
Dist)	$(A \wedge (B \vee C)) \to ((A \wedge B) \vee C)$
DNI)	$A \to \neg\neg A$ (for convenience)

MP)	$A, A \to B \vdash B$
Adj)	$A, B \vdash A \wedge B$
trans)	$A \to B, B \to C \vdash A \to C$

and not including Disjunctive Syllogism (DS) and not including *ex contradictione quodlibet* (ECQ)

DS)	$A \vee B, \neg A \vdash B$
ECQ)	$(A \wedge \neg A) \to B$

ii) Deontic extensions, **DPL** —

PL)	all of **PL** for the deontic language,
C)	$(OA \wedge OB) \to O(A \wedge B)$
RM)	If $\vdash A \to B$ then $\vdash OA \to OB$
K)	$O(A \to B) \to (OA \to OB)$
Bel)	$O(A \vee B) \to (\neg O \neg A \vee OA)$
N)	If $\vdash A$ then $\vdash OA$

Not —

DDS)	$O(A \vee B), O\neg A \vdash OB$

— The Smith Argument (from Horty) —

i) Smith ought to fight in the army or perform alternative national service. — $O(f \vee s)$

ii) Smith ought not to fight in the army. — $O\neg f$

∴ iii) Smith ought to perform alternative national service. — Os

— Summary: Non-explosive Systems —

a) Non-adjunctive systems, reject (C)

— MDL = **P** = CL + (RM) + (N)

b) Non-distributive systems, reject (RM)

— **ECN** = CL + (RE) + (C) + (N)

c) Non-aggregative, non-distributive systems, reject both (C) and (RM)

— **EN** = CL + (RE) + (N)

d) Paraconsistent systems, reject (ECQ)

— **PDL** = PL + (C) + (RM) + (K) + (Bel) + (N)

Part II

— ADAPTIVE LOGICS —

— Adaptive Logics in Standard Format —

Sailing between a lower-limit logic, **LLL**, and an upper-limit logic, **ULL**.

LLL, a base of unquestioned inferences, likely too weak for its purposes.

ULL, the standard for normal reasoning, but likely too strong when reasoning about abnormal situations.

In **PL** —

i)	$A \vee B$	PREM
ii)	$\neg A$	PREM
iii)	$\neg A \wedge (A \vee B)$	i, ii Adj
iv)	$B \vee (A \wedge \neg A)$	iii Dist

Contrast (ECQ) —

i)	A	PREM
ii)	$\neg A$	PREM
iii)	$A \vee B$	i, $\vee$-intro
iv)	$\neg A \wedge (A \vee B)$	ii, iii, Adj
v)	$B \vee (A \wedge \neg A)$	iv, Dist

Adaptive (DS) —

$$\begin{array}{ll} A \vee B & \Delta_1 \\ \neg A & \Delta_2 \\ \hline B & \{A \wedge \neg A\} \cup \Delta_1 \cup \Delta_2 \end{array}$$

— Adaptive Logics in Standard Format, in general —

The ‘Standard Format’ —

$\mathbf{AL} = \langle \mathbf{LLL}, \Omega, \text{Strategy} \rangle$ —

where Ω is a set of abnormalites specified by logical form, and the Strategy determines how abnormalities are treated.

(For present purposes Strategy = Reliability.)

General proof theory —

Definition: (Dab-formulas) — For finite $\Theta \subset \Omega$, $Dab(\Theta) = \bigvee \Theta$, the disjunction of members of Θ.

PREM If $A \in \Gamma$,

$$\begin{array}{ll} \ldots & \ldots \\ \hline A & \emptyset \end{array}$$

RU If $A_1, \ldots, A_n \vdash_{\mathbf{LLL}} B$,

$$\begin{array}{ll} A_1 & \Delta_1 \\ \vdots & \vdots \\ A_n & \Delta_n \\ \hline B & \Delta_1 \cup \cdots \cup \Delta_n \end{array}$$

RC If $A_1, \ldots, A_n \vdash_{\mathbf{LLL}} B \vee Dab(\Theta)$,

$$\begin{array}{ll} A_1 & \Delta_1 \\ \vdots & \vdots \\ A_n & \Delta_n \\ \hline B & \Theta \cup \Delta_1 \cup \cdots \cup \Delta_n \end{array}$$

a) Marking for Reliability Strategy —

i) Minimal *Dab*-formulas — $Dab(\Theta)$, is 'minimal' as of a stage s in a derivation from Γ just in case $Dab(\Theta)$ is derived within s on the empty condition $\emptyset$, and further there is no Θ' such that $\Theta' \subset \Theta$ and $Dab(\Theta')$ is derived within s on $\emptyset$.

ii) Unreliabilities from Γ — $U_s(\Gamma) = \Theta_1 \cup \cdots \cup \Theta_n$, where $Dab(\Theta_1), \ldots,$ $Dab(\Theta_n)$ are all the minimal *Dab*-formulas at a stage s.

iii) Mark line i in stage s iff the condition on i is Δ and $\Delta \cap U_s(\Gamma) \neq \emptyset$.

b) Final derivability — $\Gamma \mathrel{|\!\!\sim}_{\mathbf{AL}^r} A$ iff there is a line i in a stage s of a derivation from Γ such that A is not marked at i in s, and every extension of the proof in which i is marked has an extension in which it is unmarked.

— Adaptive Paraconsistent systems —

$\mathbf{APL}^r$ — $\mathbf{LLL} = \mathbf{PL}$; $\Omega = \{A \wedge \neg A\}$

Adaptive (DS) — $A \vee B, \neg A \mid\!\sim_{\mathbf{APL}^r} B$

i)	$A \vee B$	PREM	$\emptyset$
ii)	$\neg A$	PREM	$\emptyset$
iii)	$\neg A \wedge (A \vee B)$	i, ii RU	$\emptyset$
iv)	$B \vee (A \wedge \neg A)$	iii RU (Dist)	$\emptyset$
v)	B	iv RC	$\{A \wedge \neg A\}$

But not (ECQ)

i)	A	PREM	$\emptyset$
ii)	$\neg A$	PREM	$\emptyset$
iii)	$A \vee B$	i, RU ($\vee$I)	$\emptyset$
iv)	$\neg A \wedge (A \vee B)$	ii, iii RU (Adj)	$\emptyset$
v)	$B \vee (A \wedge \neg A)$	iii RU (Dist)	$\emptyset$
✓vi)	B	iv RC	$\{A \wedge \neg A\}$
vii)	$A \wedge \neg A$	i, ii RU (Adj)	$\emptyset$

Adaptive (DDS) — $O(A \vee B), O\neg A \mid\!\sim_{\mathbf{ADPL}^r} OB$ — with **DPL** in place of **PL**

i)	$O(A \vee B)$	PREM	$\emptyset$
ii)	$O\neg A$	PREM	$\emptyset$
iii)	$\neg O\neg A \vee OB$	i RU (Bel)	$\emptyset$
iv)	$O\neg A \wedge (\neg O\neg A \vee OB)$	ii, iii RU	$\emptyset$
v)	$OB \vee (O\neg A \wedge \neg O\neg A)$	iv RU (Dist)	$\emptyset$
vi)	OB	v RC	$\{O\neg A \wedge \neg O\neg A\}$

For example, $O(f \vee s), O\neg f \mid\!\sim_{\mathbf{ADPL}^r} Os$

i)	$O(f \vee s)$	PREM	$\emptyset$
ii)	$O\neg f$	PREM	$\emptyset$
iii)	$\neg O\neg f \vee Os$	i RU	$\emptyset$
iv)	$O\neg f \wedge (\neg O\neg f \vee Os)$	ii, iii RU	$\emptyset$
v)	$Os \vee (O\neg f \wedge \neg O\neg f)$	iv RU	$\emptyset$
vi)	Os	v RC	$\{O\neg f \wedge \neg O\neg f\}$

But (DEX) — $Op, O\neg p \mathrel{|\!\!\sim}_{\mathbf{ADPL}^r} Oq$ — !!!

i)	Op	PREM	$\emptyset$
ii)	$O\neg p$	PREM	$\emptyset$
iii)	$O(p \vee q)$	i RU	$\emptyset$
iv)	$\neg O\neg p \vee Oq$	iii RU (Bel)	$\emptyset$
v)	$O\neg p \wedge (\neg O\neg p \vee Oq)$	ii, iv RU	$\emptyset$
vi)	$Oq \vee (O\neg p \wedge \neg O\neg p)$	v RU (Dist)	$\emptyset$
vii)	Oq	vi RC	$\{O\neg p \wedge \neg O\neg p\}$

Hence, modify Ω —

ADPL^r — **LLL** = **DPL**; and

$\Omega = \{C : \exists A(C = (A \wedge \neg A) \vee (OA \wedge \neg OA) \vee (OA \wedge O\neg A))\}$

The Smith Argument succeeds—

i)	$O(f \vee s)$	PREM	$\emptyset$
ii)	$O\neg f$	PREM	$\emptyset$
iii)	$\neg O\neg f \vee Os$	i RU (Bel)	$\emptyset$
iv)	$O\neg f \wedge (\neg O\neg f \vee Os)$	ii, iii RU (Adj)	$\emptyset$
v)	$Os \vee (O\neg f \wedge \neg O\neg f)$	iv RU (Dist)	$\emptyset$
vi)	$Os \vee (\neg f \wedge \neg\neg f) \vee (O\neg f \wedge \neg O\neg f) \vee (O\neg f \wedge O\neg\neg f)$	v RU ($\vee$I)	$\emptyset$
vii)	Os	vi RC	Δ

where $\Delta = \{(\neg f \wedge \neg\neg f) \vee (O\neg f \wedge \neg O\neg f) \vee (O\neg f \wedge O\neg\neg f)\}$

(DEX) fails —

i)	Op	PREM	$\emptyset$
ii)	$O\neg p$	PREM	$\emptyset$
iii)	$O(p \vee q)$	i RU	$\emptyset$
iv)	$\neg O\neg p \vee Oq$	iii RU (Bel)	$\emptyset$
v)	$O\neg p \wedge (\neg O\neg p \vee Oq)$	ii, iv RU (Adj)	$\emptyset$
vi)	$Oq \vee (O\neg p \wedge \neg O\neg p)$	v RU (Dist)	$\emptyset$
vii)	$Oq \vee (\neg p \wedge \neg\neg p) \vee (O\neg p \wedge \neg O\neg p) \vee (O\neg p \wedge O\neg\neg p)$	vi RU ($\vee$I)	$\emptyset$
✓vii)	Oq	vi RC	Δ
viii)	$(O\neg p \wedge O\neg\neg p)$	i, ii RU (DNI, RM, Adj)	$\emptyset$
ix)	$(\neg p \wedge \neg\neg p) \vee (O\neg p \wedge \neg O\neg p) \vee (O\neg p \wedge O\neg\neg p)$	viii RU ($\vee$I)	$\emptyset$

where $\Delta = \{(\neg p \wedge \neg\neg p) \vee (O\neg p \wedge \neg O\neg p) \vee (O\neg p \wedge O\neg\neg p)\}$

— Classically based systems —

Adaptive aggregation (C) —

LLL = **P** = MDL = [CL + (RM) + (N) + (P)]

False starts —

1) $\Omega = \{OA \wedge O\neg A\}$

Problem — $\nvdash_{\mathbf{P}} (OA \wedge OB) \rightarrow (O(A \wedge B) \vee (OA \wedge O\neg A) \vee (OB \wedge O\neg B))$

2) $\Omega = \{OA \wedge OB \wedge \neg O(A \wedge B)\} \cup \{OA \wedge O\neg A\}$

since

$$\vdash_{\mathbf{P}} (OA \wedge OB) \rightarrow (O(A \wedge B) \vee (OA \wedge OB) \wedge \neg O(A \wedge B),$$

and

$$\vdash_{\mathbf{P}} (OA \wedge OB) \rightarrow (O(A \wedge B) \vee (OA \wedge OB) \wedge \neg O(A \wedge B) \vee (OA \wedge O\neg A) \vee (OB \wedge O\neg B))$$

Problem — (DEX) again!!!

i)	Op	PREM	$\emptyset$
ii)	$O\neg p$	PREM	$\emptyset$
iii)	$O(p \vee q)$	i RU (RM)	$\emptyset$
iv)	$O(\neg p \vee q)$	ii RU (RM)	$\emptyset$
v)	$O((p \vee q) \wedge (\neg p \vee q))$	iii, iv RC	Δ
vi)	Oq	v RU (RE)	Δ

with $\Delta = \{O(p \vee q) \wedge O(\neg p \vee q) \wedge \neg O((p \vee q) \wedge (\neg p \vee q))\}$.

vii)	$Op \wedge O\neg p$	i, ii RU (Adj)	$\emptyset$
viii)	$(O(p \vee q) \wedge O(\neg p \vee q) \wedge \neg O((p \vee q) \wedge (\neg p \vee q))) \vee (Op \wedge O\neg p)$	vii RU ($\vee$I)	$\emptyset$

(Although $\{(\text{viii})\} = \Delta$, (viii) not a minimal *Dab*-formula for $\Gamma = \{(\text{i}), (\text{ii})\}$. Line (vi) left unmarked.)

3) $\Omega = \{D : \exists A \exists B \exists C(D = (OA \wedge OB \wedge \neg O(A \wedge B)) \vee (OC \wedge O\neg C\}$

Problem — Also yields (DEX)!!! (Essentially the same proof)

A better account —

Notation — When $B_1, \ldots, B_n$ all the subformulas of A, then
$\mho(A) = UB_1 \wedge \cdots \wedge UB_n$, (where $UB = \neg(OB \wedge O\neg B)$

$$\vdash_{\mathbf{P}} (OA \wedge OB) \to (O(A \wedge B) \vee (OA \wedge OB \wedge \neg O(A \wedge B) \wedge \mho(A \wedge B)) \vee \neg\mho(A \wedge B))$$

$\mathbf{AP}^r$ — $\mathbf{LLL} = \mathbf{P}$, and

$$\Omega = \{C : \exists A \exists B (C = (OA \wedge OB \wedge \neg O(A \wedge B) \wedge \mho(A \wedge B)) \vee \neg\mho(A \wedge B)\},$$

or equivalently,

$$\Omega = \{C : \exists A \exists B (C = (OA \wedge OB \wedge \neg O(A \wedge B)) \vee \neg\mho(A \wedge B)\}$$

The Smith Argument —

i)	$O(f \vee s)$	PREM	$\emptyset$
ii)	$O\neg f$	PREM	$\emptyset$
iii)	$O(\neg f \wedge (f \vee s))$	i, ii RC	Δ
iv)	$O(\neg f \wedge s)$	iii RU	Δ
v)	Os	iv RU	Δ

where $\Delta = \{(O(f \vee s) \wedge O\neg f \wedge \neg O(\neg f \wedge (f \vee s))) \vee (Of \wedge O\neg f) \vee$
$(Os \wedge O\neg s) \vee (O(f \vee s) \wedge O\neg(f \vee s)) \vee (O(\neg f \wedge (f \vee s)) \wedge O\neg(\neg f \wedge (f \vee s)))\}$

— Nonmonotonicity —

In general — $\Gamma \mid\!\sim A$ but $\Gamma' \not\mid\!\sim A$, when $\Gamma \subset \Gamma'$

Extended (conflict) Smith Argument — $O(f \vee s), O\neg f, O\neg s \not\mid\!\sim_{\mathbf{AP}^r} Os$

i)	$O(f \vee s)$	PREM	$\emptyset$
ii)	$O\neg f$	PREM	$\emptyset$
iii)	$O(\neg f \wedge (f \vee s))$	i, ii RC	Δ
iv)	$O\neg f \wedge s)$	iii RU	Δ
✓v)	Os	iv RU	Δ
vi)	$O\neg s$	PREM	$\emptyset$
vii)	$(O(f \vee s) \wedge O\neg f \wedge \neg O(\neg f \wedge (f \vee s))) \vee \neg \mho(\neg f \wedge (f \vee s))$	i, ii, vi RU	$\emptyset$

Adaptive Distribution —

AECN^r — **LLL** = **ECN** = [CL + (RE) + (C) (or a weaker version) + (N)] (with a weaker (C), (P) is an option).

$$\Omega_b = \{C : \exists A \exists B(C = (O(A \wedge B) \wedge \neg OA) \vee \neg \mho(A \wedge B)\}$$

Extreme adaptation —

$\mathbf{AEN}^r$ — $\mathbf{LLL} = \mathbf{EN} = [\mathrm{CL} + (\mathrm{RE}) + (\mathrm{N})]$

$\Omega = \Omega_{\mathbf{p}} \cup \Omega_{\mathbf{ecn}}$, where

$$\Omega_{\mathbf{p}} = \{C : \exists A \exists B(C = (OA \wedge OB \wedge \neg O(A \wedge B)) \vee \neg \mho(A \wedge B)\}$$
$$\Omega_{\mathbf{ecn}} = \{C : \exists A \exists B(C = (O(A \wedge B) \wedge \neg OA) \vee \neg \mho(A \wedge B)\}$$

— Summary —

$\mathbf{ADPL}^r = \langle \mathbf{DPL}, \Omega, \text{Reliability} \rangle$, with

$\Omega = \{C : \exists A(C = (A \wedge \neg A) \vee (OA \wedge \neg OA) \vee (OA \wedge O\neg A)\}$

$\mathbf{AP}^r = \langle \mathbf{P}, \Omega, \text{Reliability} \rangle$, with

$\Omega = \{C : \exists A \exists B(C = (OA \wedge OB \wedge \neg O(A \wedge B)) \vee \neg\mho(A \wedge B)\}$

$\mathbf{AECN}^r = \langle \mathbf{ECN}, \Omega, \text{Reliability} \rangle$, with

$\Omega = \{C : \exists A \exists B(C = (O(A \wedge B) \wedge \neg OA) \vee \neg\mho(A \wedge B)\}$

$\mathbf{AEN}^r = \langle \mathbf{EN}, \Omega, \text{Reliability} \rangle$, with $\Omega = \Omega_{\mathbf{p}} \cup \Omega_{\mathbf{ecn}}$, where

$\Omega_{\mathbf{p}} = \{C : \exists A \exists B(C = (OA \wedge OB \wedge \neg O(A \wedge B)) \vee \neg\mho(A \wedge B)\}$
$\Omega_{\mathbf{ecn}} = \{C : \exists A \exists B(C = (O(A \wedge B) \wedge \neg OA) \vee \neg\mho(A \wedge B)\}$

For all, $\mathbf{ULL} = \text{SDL}$

Hence, $OA \mid\!\sim_{\mathbf{AL}^r} \neg O \neg A$, and more generally

If Γ is SDL-consistent, then $\Gamma \mid\!\sim_{\mathbf{AL}^r} A$ iff $\Gamma \vdash_{\mathbf{SDL}} A$

— Concerns —

1) (presumptively, no-conflict) — $OA \mathrel{|\!\sim}_{\mathbf{AL}^r} \neg O \neg A$

2) (nonmonotonicity) — $O(f \vee s), O\neg f \mathrel{|\!\sim}_{\mathbf{AL}^r} Os$, but
$O(f \vee s), O\neg f, O\neg s \mathrel{|\!\not\sim}_{\mathbf{AL}^r} Os$