The Koshen–Specker theorem and ontological (in)completeness of quantum objects

Raoni Wohnrath Arroyo*[1-4]

[1]Centre for Logic, Epistemology and the History of Science, University of Campinas, Campinas, SP, Brazil. Support: grant #2021/11381-1, São Paulo Research Foundation (FAPESP).
[2]Research Group on Logic and Foundations of Science, National Council for Scientific and Technological Development (CNPq), Florianópolis, SC, Brazil.
[3]International Network on Foundations of Quantum Mechanics and Quantum Information, Florianópolis group, Florianópolis, SC, Brazil.

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Abstract

Due to the Koshen–Specker theorem, quantum objects are incomplete objects regarding context-dependent properties, and that —in a specific interpretation of quantum mechanics, namely the "Consciousness Cause Collapse Hypothesis"— the transition from metaphysical incompleteness to completeness happens due to the interaction with conscious minds.

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*rwarroyo@unicamp.br

1 Incomplete objects

Arroyo and Arenhart (2019) argued that quantum mechanics, under a specific interpretation viz. the "Consciousness Causes Collapse Hypothesis" (cf. de Barros and Oas, 2017) rehabilitates the controversial substance dualism thesis in metaphysics, offering quantum-mechanical arguments for such. Underlying the argument is the idea that quantum-mechanical systems depend on the mind, so the mind cannot depend on the physical domain. So, in an important sense, substance dualism is justified by quantum mechanics (up to a specific interpretation of it). The authors put forth three metametaphysical conditions that any metaphysical profile for CCCH's mind should comply, viz.: causality, transcendence, and interaction (cf. Arroyo and Arenhart, 2019, p. 38). In this paper, I'll push the arguments further and propose a fourth metametaphysical condition to such metaphysics of science: *completeness*, due to the Koshen–Specker (KS) theorem.

The metaontological background of this discussion is what Parsons (1980, pp. 18-21) called an "incomplete object". Here's Parsons (1980):

> By calling an object 'complete,' I mean that for any nuclear property, the object has either that property or it has its negation. (Parsons, 1980, p. 19).

An incomplete object is then, by definition, an object that does not possesses certain properties nor its negation. Moreover, completeness is a feature of *existing* objects only: "[...] all existing objects are complete" (Parsons, 1980, p. 20). To be sure, let us check the following example.

> Consider the object whose properties are goldness and mountainhood. It does not have the property of blueness, nor does it have the property of nonblueness; I will say that it is indeterminate with respect to blueness. That object will in fact be indeterminate with respect to every nuclear property except goldness and mountainhood. [...][A]dd to the "the gold mountain" all nuclear properties that are entailed by

> goldness and mountainhood. Then it will have, for example, the nuclear property of either-being-located-in-North-America-or-not-being-located-in-North-America, but it will not have either of those disjuncts; it will be indeterminate with respect to being located in North America. (Parsons, 1980, pp. 20-21).

So far, so good. With that under our belts, let us steer to standard non-relativistic quantum mechanics. Take a class of indistinguishable quantum objects, such as electrons. *All* electrons have the state-*independent* properties of having the same rest mass ($0.511MeV$), electric charge ($-1.6 \times 10^{-19}C$), and spin ($\frac{1}{2}\hbar$). They're all indistinguishable objects in this sense (cf. Bigaj, 2022; French and Krause, 2006). But there are also state-*dependent* properties which *also* characterizes these quantum objects. Such properties, however, as the name suggests, depend of the state it assumes. The state-dependent properties, on its turn, are context-dependent (or *measurement*-dependent).

To see the metaphysical implications of this, let us review very briefly the state-dependent properties of quantum-mechanical objects, and how they come to possess them.

2 Collapse, contextuality, and ontological completeness

Standard quantum-mechanical descriptions are, on the one hand, deterministic in the sense that the evolution of a state $|\psi_0\rangle$ of a system in time t_0 is given by

$$|\psi\rangle = e^{-\frac{i}{\hbar}\hat{H}t}|\psi_0\rangle \tag{1}$$

which uniquely determines the state of the system at times $t \neq t_0$. On the other hand —and this was firstly pointed out by Bohr (1928)— standard quantum-mechanical descriptions are *also* probabilistic in the sense that all that we can talk about the system $|\psi\rangle$ are the probabilities P for possible measurement outcomes o_i of experiments, represented by the observable operator $\hat{O}$, such that

$$P(o_i) = |\langle o_i|\psi\rangle|^2 \tag{2}$$

and

$$\hat{O}|o_i\rangle = o_i|o_i\rangle \tag{3}$$

The problem is that both quantum-mechanical descriptions are incompatible, and they are separated by the notion of "measurement". Equation 1 admits superpositions as solutions, meaning that a standard quantum-mechanical description of a detector d ready to measure z-spin states of an electron e has a reset state —e.g. "ready"— and two indicator states concerning the states of that the pointer can assume e.g. UP and DOWN. This means that

$$|z_{\text{UP}}\rangle_e \otimes |\text{reset}\rangle_d \rightarrow |z_{\text{UP}}\rangle_e \otimes |\text{UP}\rangle_d$$

and

$$|z_{\text{DOWN}}\rangle_e \otimes |\text{reset}\rangle_d \rightarrow |z_{\text{DOWN}}\rangle_e \otimes |\text{DOWN}\rangle_d$$

If, instead, one wants to measure the eigenstate of a x-spin electron, and since

$$|x_{\text{UP}}\rangle_e = \frac{1}{\sqrt{2}}|z_{\text{UP}}\rangle_e + \frac{1}{\sqrt{2}}|z_{\text{DOWN}}\rangle_e$$

then the initial wave function of the state can be described as

$$|\psi_{t=0}\rangle = \left(\frac{1}{\sqrt{2}}|z_{\text{UP}}\rangle_e + \frac{1}{\sqrt{2}}|z_{\text{DOWN}}\rangle_e\right) \otimes |\text{reset}\rangle_d \tag{4}$$

and must evolve to

$$|\psi\rangle_{t\neq 0} = \frac{1}{\sqrt{2}}\left(|z_{\text{UP}}\rangle_e \otimes |\text{UP}\rangle_d\right) + \frac{1}{\sqrt{2}}\left(|z_{\text{DOWN}}\rangle_e \otimes |\text{DOWN}\rangle_d\right). \tag{5}$$

It is, however, hard to appreciate what kind of macroscopic state the sum of measuring states $|\text{UP}\rangle_d + |\text{DOWN}\rangle_d$ may represent. This is why Dirac (1930) posited that although the formalism describes quantum systems as superpositions, we never measure such superpositions; instead, only unique measurement results count as measurement outcomes, viz. $|\textsc{up}\rangle_d$ and not $|\textsc{down}\rangle_d$, or $|\textsc{down}\rangle_d$ and not $|\textsc{up}\rangle_d$. The debate, then, revolved around under which circumstances such transition takes place. To Bohr (1928, p. 102), this happens because of the interaction between a microscopic system and a macroscopic system. This kind of conception aroused the notion of a "boundary" between so-called "classical" and "quantum" domains of reality, in which superpositions occur in the latter but not in the former, so when a measurement is made, it allegedly brings the quantum system to the *classical* realm. Pauli (1950) coined the term "Heisenberg's cut" ("*Heisenbergscher Schnitt*") to describe such situation.

As first pointed out by von Neumann (1932), the so-called "cut" is completely arbitrary, i.e. it could be placed anywhere not just between the quantum object and the classical measurement apparatus, but anywhere between quantum systems and the observer's brain. The assumption is that if quantum mechanics holds for physical systems, it must hold for *all* physical systems —including the measurement apparatus. Moreover, it should hold for the experimenter's eye, optical nerve, and brain. More formally, the rationale is that the quantum object e, the macroscopic measurement device d, the eye and optic nerve of the observer n up to the observer's brain b are subsystems of the state Hilbert space $\mathscr{H}$, so that $\mathscr{H} = \mathscr{H}_e \otimes \mathscr{H}_d \otimes \mathscr{H}_n \otimes \mathscr{H}_b$ but *still* without presenting a single eigenvector as the result —which is to be expected as a measurement outcome. So we have:

$$\begin{aligned} |\psi\rangle = \alpha\Big(|z_{\textsc{up}}\rangle_e \otimes |\textsc{up}\rangle_d \otimes |\textsc{up}\rangle_n \otimes |\textsc{up}\rangle_b\Big) + \\ \beta\Big(|z_{\textsc{down}}\rangle_e \otimes |\textsc{down}\rangle_d \otimes |\textsc{down}\rangle_n \otimes |\textsc{down}\rangle_b\Big) \end{aligned} \tag{6}$$

Just as in equation 5, it is hard to see which macroscopic physical system equation 6 could represent. So the coupling of a measurement apparatus —or, for that matter, a *physical* observer— can be of any help. As Baggott (1992) summarizes:

> Quantum particles are known to obey the laws of quantum theory: they are described routinely in terms of superpositions of the measurement eigenstates of devices designed to detect them. Those devices are them-

> selves composed of quantum particles and should, in principle, behave similarly. This leads us to the presumption that linear superpositions of macroscopically different states of measuring devices (different pointer positions, for example) are possible. But the observer never actually sees such superpositions. (Baggott, 1992, p. 186).

This situation is called by d' Espagnat (1999, p. 169) as "von Neumann's chain". So one needs to break such a chain in order to account for measurement outcomes, viz. to account for an empirically adequate theory (cf. van Fraassen, 1991). The solution proposed by von Neumann (1932, p. 420) is that different dynamics occur when the system interacts with the observer's *abstract ego*: instead of obeying the linearity implicit in "process 2", when interacting with the observer's abstract ego the system collapses immediately to one of its eigenstates with a given probability $|\alpha|^2 + |\beta|^2 = 1$. Although the literature often emphasizes that von Neumann (1932) coined the interpretation according to which the mind causes the collapse (cf., for instance, Jammer, 1974), he indeed never mentioned the term "mind" nor "consciousness" in his work (cf. Bueno, 2019). It was Wigner (1983) who explicitly states that the agent which is responsible for the collapse is the observer's mind —which is why de Barros and Montemayor (2019) and de Barros and Oas (2017) coined the term "Consciousness Causes Collapse Hypothesis (CCCH)".

As pointed out by Arroyo and Arenhart (2019), this ontological commitment with the existence of mind demands several metaphysical constraints, viz. this mind must be *causal* (i.e. must act upon matter), *transcendent* (i.e. not reducible to matter), and *interactive* (i.e. must interact with matter). This, in its turn, places several constraints on the various kinds of metaphysical profiles one might want to associate with CCCH's mind —including several kinds of moderate dualisms.[1]

Now I mention the CCCH because there is an interesting link between the notion of "collapse" and the KS theorem —also referred as a "paradox" (Kochen, 2019, pp. 252-254).[2] Zeilinger (2005) states that the KS theorem implies that:

[1]For a comprehensive list of kinds of dualism, cf. Lycan (2009) and Rodrigues (2014).

[2]I acknowledge that there is tension on whether the notion of "contextuality", as implied by the KS

> [...] even for single particles, it is not always possible to assign definite measurement outcomes independently of and prior to the selection of specific measurement apparatus in the specific experiment. (Zeilinger, 2005, p. 743).

It is worth emphasizing that Zeilinger's remark (and my focus in this paper) is — exclusively— on *standard* quantum mechanics (cf. Cohen-Tannoudji et al., 2020; Messiah, 1961), a theory which can be interpreted as CCCH. On other reconstructions of quantum mechanics, this is *not* the case; as stressed by Daumer, Dürr, Goldstein, Maudlin, Tumulka, and Zanghì (2006, p. 131), "[i]ndeed, Bohmian mechanics and the Ghirardi–Rimini–Weber version of quantum mechanics allow us to do precisely this, since they do not postulate some special physics for measurements". In few words, as remarked by da Costa (2019), the famous theorem due to Kochen and Specker (1967) states that:

> The results observed in measurement are dependent upon what other measurements are being made; in other words, the result of a measurement of an observable is dependent on which other commutating observables are being measured. (Quantum contextuality means that the result of a measurement of a quantum observable is dependent on which other commuting observables are being regarded.) [...] *each observable of a quantum system should have a well-defined value in any instant of time, what is false according to the theorem.* (da Costa, 2019, p. 75, emphasis added).

Going even further, de Barros and Montemayor (2019) states that:

> It so happens that the idea that a superposition is a state with either one property or the other is not consistent. So, *a measurement does not reveal the existing value of a property, but seems to create it.* (de Barros and Montemayor, 2019, p. 57, emphasis added).

theorem makes reference to measurement or not. While Cabello (2021), da Costa (2019), de Barros and Montemayor (2019), Held (2018), Kochen (2019), Leggett (1991), and Ru et al. (2022) links both concepts, de Ronde (2016) disentangle them. In the remainder of this paper, I'll endorse the former as a working hypothesis.

According to Baradad (2022), one of the most striking philosophical implications of the KS theorem is:

> [...] the ineliminable contextuality of measurement; or to put it another way, the downfall of the metaphysics of individualism (the assumption that there are pre-existing individuals with a full set of determinate properties). (Baradad, 2022, p. 1044).

Finally, to close this selection, I call Leggett (1991, p. 87) which states that —in standard quantum mechanics— "[...] it is the act of measurement that is the bridge between the microworld, which does not by itself possess definite properties, and the macroworld, which does". The notion of "measurement" is ill-defined in the foundations of quantum mechanics (for discussion, cf. Arroyo and da Silva, 2021); nevertheless, according to textbook approaches (cf. Cohen-Tannoudji et al., 2020; Messiah, 1961), the act of measurement collapses/projects wave functions from superposed states into eigenstates of a measured property e.g. spin UP in a given axis (and not DOWN in the same axis).

The KS theorem, thus, —at least within the collection of views gathered above— seems to imply that *quantum objects are incomplete objects up to contextuality/measurement contexts*. That is to say that it is due to the act of measurement (which, to CCCH, is caused by the observer's mind) that quantum objects acquire completeness in a metaphysical sense. In this sense, following Parsons (1980), one might say that —prior to measurement contexts— quantum objects does not possesses the property of having a spin value of UP nor ¬UP (i.e. DOWN). They are *incomplete objects* with regards to state-dependent/context-dependent properties (such as spin, position, momentum). In fact, this idea aligns with the study conducted by Arenhart and Felippe Junior (2020), which argues that the KS theorem imposes several restrictions for the application of traditional metaphysical theories of individuality to quantum objects, viz. the bundle and substratum theory of individuality. This happens, as they argue, because of the state-dependent/context-dependent properties that quantum objects may acquire, maintaining the consistency of quantum mechanics. Such state-dependent/context-dependent properties, as I argue, are the dividing line between quantum objects' completeness and incompleteness, being the mind the agent who makes the transition. Moreover, if we stick to Parson's (1980) metaontological rule of thumb for what counts as an existing object, incomplete objects are *nonexistent* objects; hence, quantum objects comes to existence by virtue of their interaction with the mind.

Due to the KS theorem, then, I might add to Arroyo and Arenhart's (2019) list for the metaphysical constraints of CCCH's mind the following:

Completeness. It is due to the causal interaction with the transcendent mind that quantum objects acquire completeness; otherwise, their state-dependent/context-dependent properties are non-existent.

References

Arenhart, J. R. B. and H. Felippe Junior (2020), "The Fate of Bundle and Substratum Theories Under KS Theorem," in *A True Polymath. A Tribute to Francisco Antonio Doria*, ed. by J. A. de Barros and D. Krause, College Publications, Rickmansworth, pp. 1-22.

Arroyo, R. W. and J. R. B. Arenhart (2019), "Between physics and metaphysics: A discussion of the status of mind in quantum mechanics," in *Quanta and Mind*, ed. by J. A. de Barros and C. Montemayor, Synthese Library, Springer, Cham, pp. 31-42.

Arroyo, R. W. and G. O. da Silva (2021), "Against 'Interpretation': Quantum Mechanics Beyond Syntax and Semantics," *Axiomathes*, pp. 1-37, doi: 10.1007/s10516-021-09579-y.

Baggott, J. (1992), *The meaning of quantum theory: A guide for students of chemistry and physics*, Oxford University Press, New York.

Baradad, K. (2022), "Agential Realism—A Relation Ontology Interpretation of Quantum Physics," in *The Oxford Handbook of the History of Quantum Interpretations*, ed. by O. Freire Jr, Oxford University Press, New York, pp. 1031-1054.

Bigaj, T. (2022), *Identity and Indiscernibility in Quantum Mechanics*, Springer, Cham.

Bohr, N. (1928), "The Quantum Postulate and the Recent Development of Atomic Theory," *Nature*, 121, pp. 580-590.

Bueno, O. (2019), "Is There a Place for Consciousness in Quantum Mechanics?" In, *Quanta and Mind: Essays on the Connection between Quantum Mechanics and the Consciousness*, ed. by J. A. de Barros and C. Montemayor, Synthese Library, Springer, Cham, pp. 129-139.

Cabello, A. (2021), "Bell Non-locality and Kochen–Specker Contextuality: How are They Connected?" *Foundations of Physics*, 51, 61, pp. 1-13.

Cohen-Tannoudji, C., B. Diu, and F. Laloë (2020), *Quantum Mechanics, Volume I: Basic Concepts, Tools, and Applications*, trans. by S. R. Hemley, N. Ostrowsky, and D. Ostrowsky, 2nd, Wiley, Weinheim.

d' Espagnat, B. (1999), *Conceptual Foundations of Quantum Mechanics*, Perseus Books, Massachusetts.

da Costa, N. C. A. (2019), *Notas de Aula: Lógica e Fundamentos da Ciência*, Série NEL-Lógica, Décio Krause (org.), NEL – Núcleo de Epistemologia e Lógica, UFSC, Florianópolis, vol. 2.

Daumer, M., D. Dürr, S. Goldstein, T. Maudlin, R. Tumulka, and N. Zanghì (2006), "The Message of the Quantum?" *AIP Conference Proceedings*, 844, 1, pp. 129-132.

de Barros, J. A. and C. Montemayor (2019), "Between physics and metaphysics: A discussion of the status of mind in quantum mechanics," in *Quanta and Mind*, ed. by J. A. de Barros and C. Montemayor, Synthese Library, Springer, Cham, pp. 55-66.

de Barros, J. A. and G. Oas (2017), "Can We Falsify the Consciousness-Causes-Collapse Hypothesis in Quantum Mechanics?" *Foundations of Physics*, 47, pp. 1294-1308.

de Ronde, C. (2016), "Hilbert space quantum mechanics is contextual (reply to R. B. Griffiths," *Cadernos de História e Filosofia da Ciência*, 2, 1, pp. 131-160.

Dirac, P. A. M. (1930), *The Principles of Quantum Mechanics*, 1st ed., Clarendon Press, Oxford.

French, S. and D. Krause (2006), *Identity in physics: A historical, philosophical, and formal analysis*, Oxford University Press, Oxford.

Held, C. (2018), "The Kochen–Specker Theorem," in *The Stanford Encyclopedia of Philosophy*, ed. by E. N. Zalta, Spring 2018, Metaphysics Research Lab, Stanford University.

Jammer, M. (1974), *The Philosophy Of Quantum Mechanics: The Interpretations Of Quantum Mechanics In Historical Perspective*, Wiley and Sons, New York.

Kochen, S. (2019), "A Reconstruction of Quantum Mechanics," in *Philosophers Look at Quantum Mechanics*, ed. by A. Cordero, Springer, Cham, pp. 249-284.

Kochen, S. and E. P. Specker (1967), "The Problem of Hidden Variables in Quantum Mechanics," *Journal of Mathematics and Mechanics*, 17, 1, pp. 59-87.

Leggett, A. J. (1991), "Reflections on the quantum measurement paradox," in *Quantum Implications: Essays in Honour of David Bohm*, ed. by B. J. Hiley and F. D. Peat, Routledge, London, pp. 85-105.

Lycan, W. G. (2009), "Giving Dualism its Due," *Australasian Journal of Philosophy*, 87, 4, pp. 551-563.

Messiah, A. (1961), *Quantum Mechanics, Volume I*, Translated by G. M. Temmer, North-Holland, Amsterdam.

Parsons, T. (1980), *Nonexistent objects*, Yale University Press, New Heaven.

Pauli, W. (1950), "Die philosophische Bedeutung der Idee der Komplementarität," *Experientia*, 6, 2, pp. 72-75.

Rodrigues, J. G. (2014), "There are no good objections to substance dualism," *Philosophy*, 89, 2, pp. 199-222.

Ru, S., W. Tang, Y. Wang, F. Wang, P. Zhang, and F. Li (2022), "Verification of Kochen–Specker-type quantum contextuality with a single photon," *Physical Review A*, 105 (1 2022), pp. 1-8.

van Fraassen, B. C. (1991), *Quantum mechanics: An Empiricist View*, Oxford University Press, Oxford.

von Neumann, J. (1932), *Mathematische Grundlagen der Quantenmechanike*, English trans. by Robert Beyer (1955) Mathematical Foundations of Quantum Mechanics, Princeton University Press, Princeton, Springer, Berlin.

Wigner, E. (1983), "Remarks On The Mind-Body Question," in *Quantum Theory and Measurement*, ed. by J. Wheeler and W. Zurek, Princeton University Press, Princeton, pp. 168-181.

Zeilinger, A. (2005), "The message of the quantum," *Nature*, 438, p. 743.