Contradictions in reasoning: an evil or a blessing?

Walter A. Carnielli
Centre for Logic, Epistemology and the History of Science; Department of Philosophy, State University of Campinas, Brazil
R. Sérgio Buarque de Holanda 251, Cidade Universitária, Campinas, 13083-859
walter.carnielli@cle.unicamp.br

Marcelo E. Coniglio
Centre for Logic, Epistemology and the History of Science; Department of Philosophy, State University of Campinas, Brazil
R. Sérgio Buarque de Holanda 251, Cidade Universitária, Campinas, 13083-859
coniglio@cle.unicamp.br

Abilio Rodrigues
Department of Philosophy
Federal University of Minas Gerais, Brazil
Av. Presidente Antonio Carlos 6627, Fafich, Belo Horizonte, 30270-901
abilio@ufmg.br

Abstract: The purpose of this short essay is to explain the role of contradictions in the development of scientific theories. We argue that contradictions are not to be seen as a signal of defeat, but overall as a chance for improvement.

Keywords: Paraconsistency; Logics of Formal Inconsistency; Realism; Contradictions.

Is scientific realism compatible with alleged contradictions in science?

Alfred N. Whitehead (1861-1947) - the tutor of Bertrand Russell (1872-1970) at Trinity College and later his collaborator, in the first decade of the 20th century, in the well-known *Principia Mathematica* - wrote in his *Science and the Modern World* of 1925, some years after the discovery of the celebrated set-theoretical paradox by Russell: "In formal logic, a contradiction is the signal of a defeat; but in the evolution of real knowledge it marks the first step in progress towards victory" (Whitehead 1925, p. 186).

The purpose of this short essay is to explain why and how Whitehead, in his most widely read philosophic work, was basically correct in identifying the role of contradictions in the development of real (in particular scientific) knowledge, but even so, wrong on what concerns formal logic.[1] We argue that contradictions are not to be seen as a signal of defeat, but overall as a chance for improvement. Scientific realism, in simple and informal terms, is the view that the real world is genuinely described by science, so science is an actual description and not a collection of theoretical models or analogies. How can such a great description embody contradictions? We will see that the key to give a plausible answer to this question depends on how one understands contradictions.

The specialized literature recognizes as a fact the appearance of contradictions in several contexts of technical and scientific reasoning, as e.g., in computational databases, behind semantic and set theoretic paradoxes and in scientific theories. In present-day philosophy the idea that contradictions appear in several situations became almost common sense: pieces of information contradict one another, norms, laws and regulations conflict, databases clash and scientific theories collide. In the present paper we review some issues relative to the occurrence of contradictions in scientific theories, emphasizing the view that such contradictions should not necessarily be considered as possessing an ontological, but rather an epistemological character. In addition, we argue that such contradictions might be perfectly supported by a cautious logic, more prudent than classical logic.

Examples of alleged contradictions are abundant ever since the so-called rebirth of the sciences during the period from Galileo Galilei (1564-1642) to Isaac Newton (1643-1727). Nevertheless, some authors maintain that just as great as that scientific revolution, a second scientific revolution took place in the first years of the 20th century with Max Planck (1858-1947), Albert Einstein (1879-1955), Niels Bohr (1885-1962), Werner Heisenberg (1901-1976), and many others, also producing contradictions. Examples of contradictions in science involve, therefore, the disputes in the beginning of calculus involving Newton and Gottfried W. Leibniz (1646-1716), issues in Bohr's theory of the atom and classical electrodynamics. Well-known cases are Wolfgang Pauli's (1900-1958)

[1] This paper draws heavily from recent work by the authors and their collaborators.

derivation of a contradiction from Bohr's postulates and from Paul Ehrenfest's (1880-1933) adiabatic principle, and the several kinds of alleged contradictions in Newtonian cosmology. In addition to being explicitly contradictory, a scientific theory might be regarded as contradictory in many other ways, as analyzed by Colyvan (2008): some assumptions might conflict with other parts of the theory or with other well-established theories, or the theory might use different, incongruent mathematical tools. In physics, for instance, sometimes two consistent theories yield contradictions when they are put together (da Costa & French 2003, chapter 5; Meheus 2002 pp. vii-ix).

At this point it is worth to review the kind of impact that contradictions might bring to traditional logic by briefly reminding two widely known logic principles:

(i) Law of non-contradiction: no proposition *A* can be true and its negation also true, or it is impossible for one same thing to be and not to be at the same time, for example, 'an atom is not an atom'. Aristotle declared the law of non-contradiction to be the first and most certain law of reasoning.

(ii) Another principle, introduced much later in logic, called principle of explosion (*ex falso quodlibet*, *ex contradictione sequitur quodlibet* or principle of Pseudo-Scotus), says that any statement can be proven from a contradiction. Explosion is surely independent from the traditional logical principles.

According to the principle of explosion, once a contradiction is asserted, any proposition (or its negation) might be inferred from it, thus reaching what is said to be deductive triviality. This principle is very useful in mathematics, in the form of *reductio ad absurdum*, a common form of argument that seeks to demonstrate that a statement is true by showing that an absurd follows from its denial. For instance, one can prove that there is no smallest positive rational number. Argument: suppose there was one; then it could be divided by two to get a smaller one - contradiction. Hence the supposition is false. As contradictions thus imply triviality within the framework of traditional (or classical) logic, the need of scientists for a non-explosive logic to give a sensible account of contradictions seems obvious.

Throughout the history of philosophy contradictions were sometimes understood as ontological and sometimes as epistemological. But why do contradictions appear? Is there a concept or a scientific object inherently contradictory, which cannot be described in any other way than by means of a contradiction? Our basic point is that nobody knows that for sure, however, there is neither any reason nor need to start from such a formidable ontological commitment. Without supposing that a 'real contradiction' exists, simple and familiar reasons may explain the engendering of (the majority, at least) of contradictions: multiple sources, for example, if a theory is built by several authors, misrepresentation of concepts or ideas, moving from one formalism to another, or polysemy, in the sense of a same name referring to different concepts with contradictory definitions.

Regarding contradictions as epistemic helps to illuminate several difficult points in scientific reasoning. Saatsi (2014) recognizes that there are, indeed, contradictions in science, and that they give rise to many philosophical questions that might be approached from different perspectives. According to him, a useful distinction was established between logic-driven and content-driven perspectives on such contradictions with regard to the following essential question: '*Qs*: How should one explain the empirical success of inconsistent theories/models?' Besides a somewhat vague, so-called content-driven approach, Saatsi emphasizes a purely logic approach to the question:

> The logic-driven approach suggests that in response to the 'logical explosion' that results from inconsistency in classical logic, we should turn to non-classical, paraconsistent logics in order to capture (or to represent, or model, perhaps) the constraints that there must have been on scientists' inferences from an inconsistent set of assumptions. This arguably avoids the 'logical anarchy', and it may provide (part of) an answer to *Qs*. (Saatsi 2014, p. 2493)

We would like to make clear why this logic-driven view to that basic question really points to the right direction by considering that 'logical explosion' is the real issue, and not anything related to the law of non-contradiction, as mistakenly maintained by a few authors. It remains, of course, to understand what such contradictions in science really are, and what they mean. If contradictions are epistemological (and do not belong to reality), it has been already argued (see Carnielli & Rodrigues 2014) that the rejection of explosion by paraconsistentists is analogous to the rejection of the excluded middle by intuitionists. In both cases, the formal system may be understood as having not only an epistemological character, but also a character that puts a descriptive and a normative approach to logic together. This epistemological route, we claim, is of natural and immediate interest for a correct understanding of the *modus operandi* of science.

This epistemic approach does not mean, of course, that there are no ontologically contradictory scientific phenomena or objects, say in physics, chemistry or biology. What we mean to say is that our approach to paraconsistency *does not need* that presupposition: if there were real contradictions in a given science, then that science would have to develop special fields of its own, say, 'physics of contradictory particles' or 'genetics of contradictory organisms'. But this is not what we have witnessed ever since the rise of modern science. What we witness in the absolute majority of cases, without any reasonable doubt, are contradictory theories, not contradictory phenomena or objects. In this way a useful, applicable account of contradictions involving entities like propositions, theories or hypothesis might begin from the notion of evidence, a notion weaker than the notion of truth. From this point of view, if one knows that a proposition A is true, one has evidence that A is true, but not the converse: there may be some available evidence for A, even if A is not true. Following the common understanding in the philosophical world - although with some disagreement, see e.g. Achinstein (2003) - evidence for A should be understood as reasons for a justified belief in that A is true. Some examples of real situations will help to illustrate this approach. We will see that dealing rationally with contradictions plays a sound role in the organization and the acquisition of scientific knowledge.

Phlogiston and ether: useful contradictions

There are several examples of scientists holding contradictory propositions and ending up by making good profit of them. The 19th-century controversy around the movement of the luminiferous ether (Born 1956) is an exemplary case of such a situation. Experiments suggested that the ether could perform some kinds of motion relative to matter, and thus the speed of light ought to depend on the speed of this motion. The attempts to establish the theory of special relativity led physicists to confront two seemingly contradictory experiments related to such ether wind: one performed in 1885 was an apparent confirmation of Augustin-J. Fresnel's (1788-1827) stationary ether, while another conducted in 1887 was an apparent confirmation of George Stokes' (1819-1903) dragged ether. However, rather than being rejected as non-sense, such contradictory evidences influenced Einstein's views on electrodynamics and on the principle of the constancy of light. Einstein himself stated that such experiments, as well as Hendrik Lorentz's (1853-1928) theory of 1895 had influenced his thinking. He borrowed, as he said, that principle from Lorentz's stationary ether - which implies, in particular, the constancy of light in the ether theory - but recognized that this principle, together with the one of relativity, made references to the ether unnecessary. As Einstein wrote in 1907 and later on, the apparent contradiction between those principles led to certain adaptations to Lorentz's notion of local time, which had proved to be very fruitful.

The role of contradictory theories may even have a salutary effect on the expected reconciliation between Einstein's general relativity and quantum mechanics, as put by Worrall, who refers contradictory theories as inconsistent theories:

> The important point, then, is that while two mutually inconsistent theories cannot of course both be true, they may both be approximately true – that is, both may emerge as (of course different) limiting cases of some further, superior theory, just as current physicists expect General Theory of Relativity and Quantum Mechanics both to emerge as limiting cases from the eventual 'synthesis'. (Worrall 2011, p. 160)

Our position agrees with the one of Colyvan, who claims that inconsistency in science is a topic that "realists of all stripes would be well advised to think more about":

> My tentative conclusion is that anyone persuaded by the indispensability argument for scientific and mathematical realism, should also (perhaps reluctantly) sign up for belief in inconsistent objects. Note that this is not an argument that the world is inconsistent or that the world contains inconsistent objects, just that there were times when we had warrant to believe in such inconsistent objects. (Colyvan 2008, p. 119)

As much as the contradictions around the ether in physics might be seen as beneficial, the contradictory nature of the phlogiston was far from harmful to chemistry. The theory of phlogiston, an 'imponderable' substance the properties of which could not be detected through the senses, was of great help in the beginnings of chemistry in the 18th century, because it made the rise of a theory to explain combustion possible. The two main opponents, Joseph Priestley (1733-1804) and Antoine-L. Lavoisier (1743-1794), quarreled about the phlogiston for years. Lavoisier started in 1772 a series of experiments to convince the chemical community that the concept of phlogiston was superfluous. Priestley conducted similar experiments, arriving to conclusions in contradiction with Lavoisier's. One of the crucial experiments, dated from 1774, was the thermal decomposition of red oxide of mercury, which produced an 'eminently respirable air' (Holmes 1987, p. 137) able to support combustion and living organisms better than ordinary air. To Priestley, this gas was 'dephlogisticated air' – ordinary air deprived of phlogiston – the reason being that according to the then prevalent view was that fire burns out and animals die in an enclosed space because the air becomes saturated with phlogiston. To Lavoisier, this gas was oxygen (Labinger & Weininger 2005).

A more serious problem was that phlogiston theory did not only lead to contradictory conclusions, but was itself contradictory. Indeed, the calx formed when certain metals, such as magnesium, burn weighs more than the metal from which it is formed, but the contemporary hypothesis was that burning substances lost phlogiston. Supporters of the phlogiston theory put forward the additional hypothesis that metallic phlogiston had negative weight, while other combustibles contained phlogiston with positive weight - perhaps a not so absurd idea, given the alleged imponderability of phlogiston, but suspicious enough to begin convincing the chemical community of the superiority of the oxygen theory of combustion. The story was of course much more complicated, and theories of dephlogisticated air rivaled with the oxygen theory, while also other experimental results contributed to invalidate the phlogiston hypothesis. Those facts notwithstanding, the non-existence of such a hypothetical entity as the phlogiston remained difficult to demonstrate (Labinger & Weininger 2005).[2] For our purposes it is worth to remark that the attitude of those 18th century scientists regarding the many contradictions found obviously had an epistemic nature: they concentrated on the difficulties in the relevant experiments, while no one ever

[2] An analogous difficulty is the one posed by the demonstration of the non-existence of dialetheias, *qua* ontologically true contradictions. Dialetheias may be useful as hypothetical entities to simplify the explanation of paraconsistent logic, but they are hardly necessary.

concluded that fire burns and does not burn, or anything alike. In any case, the phlogiston theory, even if contradictory, gave great impetus to chemistry:

> The picture that emerges from even this abbreviated history is that of two competing theories, one of which eventually prevailed by a steady accretion of new experimental results linked to a more coherent explanatory framework. Lavoisier's balance sheet method, which accounted for the weights of all reactants and products and showed that nothing ponderable was created or destroyed, was very effective in converting other chemists to his system. Yet the rigorous application of this method required Lavoisier to adopt some premises of the scheme he had set out to overthrow. For example, in order to account for the three states of matter and for the heat released in oxidation reactions, Lavoisier posited that all substances contain varying amounts of caloric, the material basis of heat. Since he obtained nearly perfect agreement between the weights of reactants and products – to the extent that some of his opponents had understandable doubts about his claim - Lavoisier found it necessary to assume that caloric, although real, was an imponderable fluid. Shades of phlogiston!" (Labinger & Weininger 2005, pp. 1918-1919)

The phlogiston theory was the first chemical theory (as opposed to the predominant mechanical theory) that could explain the qualitative changes characteristic of chemical processes:

> A single assumption was made, that all combustible substances are alike in one respect, namely, in containing combined fire, or phlogiston [...] But many of the subsidiary hypotheses which were required to make the theory cover the new facts were contradictory, or at any rate seemed to be contradictory. (Muir 1902, chapter X)

Contradictory as it might have been, the phlogiston theory was useful and decisively contributed to found modern chemistry. And its shadows are still with us.

Contradictions do not destroy paraconsistent logic

Although in ideal terms the process of scientific investigation is supposed to act on totally coherent and complete information, in the common reality and even in the most sophisticate laboratories people have to reason based on incomplete and incorrect information, which leads to defeasible conclusions, or conclusions that have to be retracted and changed later on.

Paraconsistent logics are non-explosive, in the sense that one does not derive an arbitrary conclusion from a contradiction, because the abovementioned principle of explosion - from a pair of propositions A and not A anything follows - does not hold. A theory whose underlying logic is $A, \neg A \nvdash B)$ paraconsistent can, thus, be contradictory without being trivial. A logical system does not need to endorse the principle of explosion. Paraconsistent logic adopts a more cautious strategy: a logical consequence relation ⊢ is said to be paraconsistent when it is not explosive. Relatedly, the notion of consistency does not need to be defined in terms of freedom of contradiction. Thus, contrary to what Whitehead (1925) maintains, not even in formal logic is contradiction a signal of a defeat, and almost one century later, logical theories dealing with contradictory judgments became current.

Nevertheless, a central question remains: how do we understand the contradictions with which paraconsistent logics deal? This question is closely related to two central problems of the philosophy of logic: (i) the nature of contradictions – are they are ontological or epistemological?, and (ii) the nature of logic – is logic about language, thought or reality? Those are, as expected, open issues. With respect to the first question, two traditional positions point to opposite directions. According to Immanuel Kant (1724-1804), contradictions do not belong to reality, but are produced by reason. They arise when reason outstrips its limits – for example, when human reason tries to apprehend the infinite. They are a sign of error and must be avoided. To Georg F.W. Hegel (1770-1831), as in his *Lesser Logic*: "every actual thing involves a coexistence of opposed elements. Consequently to know, or, in other words, to comprehend an object is equivalent to being conscious of it as a concrete unity of opposed determinations" (Hegel 1975, §48).

As concerns the second question, the position we want to adopt here is that paraconsistent logic is not about reality, but about language and thought.

That logic can play both a normative and a descriptive role is an old idea in philosophy. The *Port-Royal Logic*, from the 17th century states:

> Logic is the art of conducting reasoning well in knowing things […] [Logic] serves three purposes. The first is to assure us that we are using reason well […] The second is to reveal and explain more easily the errors or defects that can occur in mental operations. The third purpose is to make us better acquainted with the nature of the mind by reflecting on its actions. (Arnauld & Nicole 1996, p. 23)

Accordingly, logic is more than a theory concerned with only preservation of truth. Paraconsistent logics, in particular the logics of formal inconsistency (LFIs) that are examined in the next section, might be seen as theories of logical consequence that combine a descriptive with a normative aspect, and allow one to reason about reasoning; they might be approached from an epistemological perspective.

LFIs are a family of propositional and quantified paraconsistent logics that encode consistency (and inconsistency) as operators independently from negation in their object language. Encoding consistency and inconsistency in this way enables one to explicitly separate not only inconsistency from triviality and non-triviality from consistency, but also contradiction from inconsistency and consistency from non-contradiction.

The language of LFIs allows investigating contradictory theories without assuming that they are necessarily trivial. LFIs reject the principle of explosion in the presence of a contradiction, unless the contradictory sentence is taken to be consistent. The family of LFIs comprises a large number of paraconsistent systems of various sorts, including the well-known hierarchy of logics introduced by Newton da Costa in Brazil in the 1960s.

The basic idea behind LFIs is that assertions about the world may be divided in two categories: consistent sentences and non-consistent sentences. Consistent propositions are subjected to classical logic, and consequently a theory T that contains a pair of contradictory sentences A and $\neg A$ explodes only when A (or $\neg A$) is taken to be a consistent sentence, linguistically marked as $\circ A$ (or $\circ\neg A$). This is the only essential distinction between LFIs and classical logic, albeit with far-reaching consequences: classical logic is expanded in such a way that in most cases a LFI is able to encode classical logic. As it was remarked, LFIs have resources to express the notion of consistency within the object language by means of a sentential unary connective called 'ball': $\circ A$ means that 'A' is consistent. As in any other paraconsistent logic, explosion does not hold. But LFIs are handled in a way that allows distinguishing between contradictions that can be accepted from those that cannot. The point of this distinction is that no matter the nature of the contradictions a paraconsistentist is willing to accept, there are contradictions that cannot be accepted. In LFIs, negation is explosive only with respect to consistent formulas (that is, formulas that are taken to be consistent):

$$\alpha, \neg\alpha \nvdash_{LFI} \beta, \text{ while } \circ\alpha, \alpha, \neg\alpha \vdash_{LFI} \beta.$$

A LFI is thus a logic that separates the sentences for which explosion holds from those for which it does not hold, being the former marked with o. For this reason, they are called *gently explosive*. Paraconsistent logics, LFIs in particular, have a good number of applications in theoretical computer science, including artificial intelligence, description logics, programming languages, software engineering and databases and information systems engineering, just to name the most popular. But is the paraconsistent paradigm relevant in the natural sciences? The answer is an emphatic 'yes', when one considers the diversity of cases that involve dramatic theoretical changes that makes scientists and mathematicians entertain contradictory theories, even if temporarily. A model of scientific rationality is outlined by Bueno and da Costa (2007) in which the role of such contradictory theories is made explicit. If we want to understand how science develops and how theories are selected and replaced in the chronology of scientific practice, one of the main problems concerning scientific rationality, according to Bueno and da Costa (2007, p. 383) is: "how to accommodate cases of scientific change where lack of consistency is involved?"

The route taken by Bueno and da Costa (2007) is to consider scientific theories as quasi-true instead of plainly true, following a previous 'theory of quasi-truth' championed by da Costa and colleagues long ago. In this way, the underlying logic is necessarily paraconsistent, thus allowing one to avoid the risk of falling into trivialism, a disaster to be eschewed at all cost, because trivial theories make no difference. Indeed, at this point the distinction between contradiction and triviality endorsed by the LFIs proves to be crucial: a contradictory theory might be informative, while a trivial theory is irrelevant.

Our interpretation of paraconsistent logic allows one to approach contradictions from an epistemological point of view as 'conflicting information', a provisional state that might be eliminated by further investigation. This does not mean that contradictions are 'desirable'; they might simply be unavoidable. Thus, reasoning under contradictions is an indispensable criterion of rationality. This view, however, does not imply that reality is contradictory in any sense: the presence of contradictions might be considered as a step in the process of acquiring knowledge that might be revised, at least in principle. But in order to be able to deal with contradictions in such a way, one needs, as it should be clear at this point, a logic that does not collapse in the face of a pair of propositions A and $\neg A$. LFIs are well suited to express this idea.

An epistemological interpretation of LFIs

A logic has an epistemic (rather than an ontological) character when its subject matter is not only the concept of truth, but also some concept strictly related to reason. This is the case of intuitionistic logic, which is concerned with truth attained in a specific way, namely, by means of a constructive proof. We claim that this epistemic approach to logic is also a way to understand paraconsistency in general, and LFIs in particular. The latter, we may say, are concerned with truth, since classical logic can be recovered for the class of consistent propositions, but they are also concerned with a notion weaker than truth, which is precisely what that allows for an intuitive and plausible understanding of the acceptance of contradictions within some contexts of reasoning.

Intuitionistic logic is a special case of paracomplete logics, that is, logics in which there is a model M and a sentence A such that both A and its negation do not hold in M. Mathematicians naturally deal with lack of information, in the sense that there are many unsolved mathematical problems. This is one of the reasons for the rejection of the excluded middle by intuitionistic logic. In the empirical sciences, although many things are obviously not known, researchers sometimes also deal with conflicting information, and very often with contradictory information. Thus at times they might have to provisionally consider two contradictory claims, one of which will be rejected in due time. Both intuitionistic and paraconsistent logics might be conceived as descriptive theories of logical consequence with epistemic character. To be sure, the fact that we find a kind of duality in the motivations for intuitionistic and paraconsistent logics is no reason for surprise (see Brunner & Carnielli 2005).

Although classical (propositional and first-order) logic is a powerful tool for modeling reasoning, its weakest point is that it does not handle contradictions in a sensible way, due to the principle of explosion. In essence, classical logic is too radical in the presence of contradictions. Contradictory information does not only occur often, but also might play a significant role in human thought, which in some cases is not entirely undesirable. As Blaise Pascal (1623-1662) stated:

> Contradiction is a bad sign of truth; several things which are certain are contradicted; several things which are false pass without contradiction. Contradiction is not a sign of falsity, nor the want of contradiction a sign of truth. (Pascal 2012, p. 84)

Indeed, in some cases things that are false can pass without contradiction. Finding contradictions in judicial testimonies, in statements from suspects of a crime or of tax fraud might be an efficient strategy: if everybody lies, there is no contradiction, and the truth will never be found. In theoretical computer science, for example, facts and rules of knowledge bases, as well as integrity constraints, might produce contradictions when combined, even when they are sound when separate. Reasoning with inconsistent data is an important research topic in the Semantic Web, as the integration of data on the web may easily lead to contradictions.

For this reason, the development of paraconsistent tools turned out to be an important issue in computer science and information systems (see e.g. Carnielli, Marcos & de Amo 2000; Decker 2006). To consider contradictions (that is, cases in which A and $\neg A$ simultaneously hold) from an epistemic viewpoint is to interpret $v(A) = v(\neg A) = 1$ as meaning that there is simultaneous, albeit non-conclusive, evidence that A is true and A is false. But what does 'evidence' mean? Evidence embodies some sense of causality, perhaps a sense of indirect causality. Although here we are not attempting to provide a definition of evidence, we might say, grossly speaking, that evidence is that which justifies belief.[3] As in Sinclair (2014, p. 315) states: "Philosophers have claimed that

[3] For a more thorough discussion see Achinstein (2003).

evidence provides justification or reasons for belief, that being rational involves respect for evidence, and that evidence serves as a neutral arbiter between competing theories, thereby maintaining scientific objectivity".

What we can say for sure is that evidence is an epistemological notion connected to ideas on knowledge and belief and to the ways how belief might be attained and knowledge acquired. Since beliefs might be contradictory, to have evidence for contradictory statements is not irrational. However, if we might accept that beliefs can be contradictory, this is not the case of knowledge. Here the consistency operator of LFIs (as mentioned before, expressed by a unary connective $\circ$ whereby $\circ A$ means that A is consistent) plays its essential role: an intuitive interpretation for $\circ A$ is that the truth value of A has been conclusively established. Understood from this point of view, a LFI is not only about truth, but simultaneously also about truth, evidence and conclusive evidence. A semantic clause for $\circ$ may be:

$$v(\circ A) = 1 \text{ implies } v(A) = 0 \text{ or } v(\neg A) = 0.$$

Notice, however, that when $v(\circ A) = 1$, that is, when the truth value of A was conclusively established, a conflicting scenario is impossible: in this case, $v(A) = 0$ or $v(\neg A) = 0$, or in other words, evidence for A (or for $\neg A$) is cancelled.

To be sure, we may think of quantifying evidence by assigning different weights to evidences of different nature, or attributing probability to distinct evidence. This is a related, but different problem, and a paraconsistentist approach to evidence might also be developed from a probabilistic point of view. Degrees of uncertainty about an event can be quantified by the maximum and minimum probabilities of that event. Because contradictions are involved, this may require a different notion of probability, as in the paraconsistent theory of probability presented by Bueno-Soler & Carnielli (2014), where the authors investigate a paraconsistent approach to probability theory, as well as a notion of paraconsistent updating through a version of Bayes' theorem for conditionalization. However, the deep connections between evidence theory and probability theory (and therefore with Bayesian conditionalization) must still be further studied.

It is worth noting that according to the above mentioned approach, if a statement A is contradictory, it cannot be consistent: if both $v(A) = 1$ and $v(\neg A) = 1$, we necessarily have $v(\circ A) = 0$. In intuitive terms, this means that contradictory evidence about A excludes conclusive evidence about A. But the converse is not true: we might have $v(\circ A) = 0$ without having both $v(A) = 1$ and $v(\neg A) = 1$. In intuitive terms, if we do not have conclusive evidence about A this does not necessarily mean that we have conflicting evidence about A.

As a consequence of this interpretation, it becomes clear that contradiction implies inconsistency, although inconsistency does not imply contradiction. This represents a radical departure from the 'traditional orthodoxy' of da Costa's view on paraconsistency, in which the notions of inconsistency and contradiction coincide.

There are still other approaches to paraconsistency, like 'dialetheism', which claims that there are true contradictions in the world. Our view differs from that approach, as it does not require legitimating the existence of any 'real contradictions'. This does not mean to say that the world has been exorcized from real contradictions - notice, however, that no one has yet found any, except perhaps for the well-known liar paradox, in the form 'This sentence is not true'. If the latter sentence, or any other of this kind, represents an ontological or epistemological contradiction is debatable, and we do not intend to solve this issue here. What we might conclude is that in a surveying account, LFIs are able to express contexts of reasoning in which contradictions occur either because reality is inherently contradictory, or because contradictions are provisional states that might be corrected later or remain in a dubious state forever. But the relevant point is that contradictions, even if not anything desirable, are not evil: just like fever indicates a hidden disease, contradictions indicate hidden difficulties in a scientific theory – and also like fever, we cannot just eradicate them under risk of catastrophic consequences. While enabling humans and machines to deal with contradictions no matter whether they are understood epistemologically or ontologically, LFIs represent a useful reasoning tool.

Now, let us return to the question presented in the first section of this paper: how can a correct description of the world embody contradictions? Actually, the answer depends on whether one considers that contradictions in scientific theories are ontological or epistemological. If one takes the former view, the compatibility between scientific realism and contradictions in empirical sciences is not a big problem. On the other hand, the view, endorsed by us, according to which contradictions are epistemic, implies that if an empirical theory embody contradictions, it cannot be considered, strictly speaking, a correct description of the world but, rather, a tool that may be useful to give solutions to specific problems.

Acknowledgements

The authors acknowledge the support of the São Paulo Research Foundation (FAPESP) grant #2010/51038-0, Thematic Project LogCons; Center of Research in Information Architecture (Centro de Pesquisa em Arquitetura da Informação), University of Brasília (CPAI/UnB); the National Council of Scientific and Technological Development (CNPq), Brazil, through several research grants; Fundação de Amparo à Pesquisa do Estado de Minas Gerais (FAPEMIG research project 21308).

References

ACHINSTEIN, P. *The Book of Evidence*. Oxford: Oxford University Press, 2003.

ARNAULD, A.; NICOLE, P. *Logic or the Art of Thinking*. Cambridge: Cambridge University Press, Cambridge, 1996 (1662).

BORN, M. (1956). *Physics in My Generation*. London: Pergamon Press, 1956.

BRUNNER, A.B.M.; CARNIELLI, W. A. "Anti-intuitionism and Paraconsistency." *Journal of Applied Logic*, 3, no. 1, pp. 161–184, 2005.

BUENO, O.; DA COSTA, N.C.A. (2007). "Quasi-truth, Paraconsistency, and the Foundations of Science". *Synthese*, 154, no. 3, pp. 383–399, 2007.

BUENO-SOLER, J.; CARNIELLI, W.A. "May Be and May Be Not: Paraconsistent Probabilities from the LFI Viewpoint. *CLE e-PRINTS,* 15, no. 2, 2015, posted June 2 2015; available at http://www.cle.unicamp.br/e-prints/vol_15,n_2,2015.html

CARNIELLI, W.A.; MARCOS, J.A. & DE AMO, S. "Formal Inconsistency and Evolutionary Databases". *Logic and Logical Philosophy*, 8, pp.115–152, 2000.

_____.; RODRIGUES, A. "Towards a Philosophical Understanding of the Logics of Formal Inconsistency". *Manuscrito* [forthcoming].

COLYVAN, M. "The Ontological Commitments of Inconsistent Theories". *Philosophical Studies*, 141, no. 1, pp. 115–123, 2008.

DA COSTA, N.A.; FRENCH, S. *Science and Partial Truth: A Unitary Approach to Models and Scientific Reasoning*. Oxford: Oxford University Press, 2003.

DECKER, H. "A Case for Paraconsistent Logic as Foundation of Future Information Systems. In: J. Castro. And E. Teniente (eds.) (2006), pp. 451-461.

HARMAN, G.; LEPORE, E. *A Companion to W.V.O. Quine*. Malden, MA: Wiley Blackwell, 2014.

HEGEL, G.W.F. *Hegel's Logic: Part One of the Encyclopaedia of the Philosophical Sciences (1830)*. Oxford: Oxford University Press, 1975.

HOLMES, F.L. *Lavoisier and the Chemistry of Life: An Exploration of Scientific Creativity*. University of Wisconsin Press, 1987.

LABINGER, J.A.; WEININGER, S.J. "Controversy in Chemistry: How Do You Prove a Negative? The Cases of Phlogiston and Cold Fusion". *Angewandte Chemie International Edition,* 44, pp. 1915-22, 2005.

MEHEUS, J. "Preface". In: J. Meheus (ed.) (2001), pp. vii-ix.

_____. *Inconsistency in Science*. Dordrecht: Springer, 2001

MUIR, M.M.P. *The Story of Alchemy and the Beginnings of Chemistry*. London: Hodder & Stoughton, 1902, in Project Gutemberg, http://www.gutenberg.org/files/14218/14218-h/14218-h.htm.

PASCAL, B. *Pensées*. CreateSpace Independent Publishing Platform, 2012.

SAATSI, J. "Inconsistency and Scientific Realism". *Synthese*, 191, no. 13, pp. 2941–2945, 2014.

SINCLAIR, R. (2014). "Quine on Evidence". In: G. Harman and E. Lepore (eds.) (2014), pp. 350-372.

WHITEHEAD, A.N. *Science and the Modern World*. New York: Macmillan Company, 1925.

WORRALL, J. "Underdetermination, Realism and Empirical Equivalence". *Synthese*, 180, no. 2, pp. 157–172, 2011.